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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

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PRESSURE EFFECTS IN STELLAR SPECTRA

By OTTO STRUVE

ABSTRACT

In order to investigate pressure broadening and pressure shift due to mol-electric Stark effect a number of *stars were selected in which the lines of singly ionized carbon and of doubly ionized silicon are perfectly sharp.* These lines are known not to be very susceptible to Stark effect.

In the spectra of these stars the *members of the "diffuse" series of helium are* invariably *hazier than the* members of *other helium series.* This is contrasted with the fact that in the laboratory the "diffuse" lines can be made sharp and narrow by reducing the pressure, but that under higher pressures they tend to be hazy, thus justifying their name. *The difference in haziness* of a "diffuse" line and any other helium line is designated as ΔH. This quantity *is variable;* several stars were found where ΔH is very small, but in the majority of cases it is quite appreciable. It appears that the value of ΔH is correlated with the width of the wings of the hydrogen lines and with the presence of the forbidden helium line at λ 4470. *All three criteria seem to show that the Stark effect is more pronounced in dwarfs than in giants.*

The *identification of the unknown line at λ 4470 with a forbidden line of helium* is discussed. The wave-length of the stellar line is 4469.84 I.A. and of the forbidden line—4469.92 I.A. (for zero field). The difference indicates an average field of 1.5×10^3 volt/cm.

Evidence concerning *other forbidden lines of helium* is collected. The *unsymmetrical contour of λ 4388 and λ 4922,* noted by Elvey, may be due to such components. *Two unidentified lines* measured by Baxandall in 24 γ Orionis *agree with two forbidden helium lines.* There is *a violet component to the line at λ 4026* which *agrees with a forbidden line.*

The *wave-lengths of certain helium lines* were measured. They *show differences* from the laboratory values, *agreeing* in sign and in size *with the pressure shifts due to unsymmetrical Stark broadening.*

There is a suspicion that the *singly ionized nitrogen lines also show a slight effect of pressure* in their appearance. The haziness of several lines, notably those at λ 4237 and λ 4242, was first noted by Baxandall.

The *intensities of the helium lines* in stellar spectra display a tendency *at first* to *increase toward the series limit and then rapidly to decrease.* In the laboratory, under ordinary conditions, they always decrease, but certain experiments by Merton and Nicholson reproduce the effects observed in the stars. The physical interpretation is lacking, but it is possible that pressure is responsible for the observed distribution of intensities. There are marked changes in the relative intensities for various stars.

The *broadening of the lines* due to Stark effect *influences the observed maxima* of

the quantity $(\lambda - \lambda_0)$ for a given value of the intensity on the line contour. If the pressure is a function of spectral type, the maximum development of the wings will not, in general, coincide with the maximum of the central intensities. A rough computation shows that *a displacement in the two maxima, corresponding to a few tenths of a spectral class, may be expected* for elements that are susceptible to Stark effect.

I. INTRODUCTION

Practically all early work on the determination of pressures in the reversing layers of the stars was based upon the observed shifts in wave-length and changes in width of certain spectral lines that were known from laboratory results to be subject to the effect of pressure. A large number of these results were erroneous, owing to our lack of familiarity with the behavior of spectral lines in the atmospheres of the stars. With the advent of the theory of ionization our ideas concerning pressures in the reversing layers have undergone a tremendous change: The numerical values were reduced from several atmospheres to thousandths or millionths parts of an atmosphere. In consequence, the earlier methods seem to have fallen into disrepute, and the opinion is now frequently heard that pressures in stellar atmospheres are so low that they do not to a measurable degree affect the appearance or the position of stellar absorption lines.[1]

It appears that in rejecting the methods based upon line width and wave-length we have gone too far. In an earlier paper[2] I have shown that there is good evidence that the widening of the lines of hydrogen and helium is principally due to mol-electric Stark effect, which in itself is directly dependent upon the pressure in the stellar atmosphere. This result can now be strengthened and enlarged by further evidence. It turns out that the spectra of stars of early type show a number of effects clearly dependent upon pressure. While it is not yet possible to derive numerical values, it is possible to determine, qualitatively, relative pressures for a number of stars. It is expected that this method will prove useful in the determination of absolute magnitudes of B-type stars.

In order to observe the effects of Stark resolution of stellar absorption lines it is necessary to select a number of stars devoid, as nearly as possible, of the effect of rotation. An inspection of the

[1] See, e.g., K. Graff, *Grundriss der Astrophysik*, p. 164, 1928.

[2] *Astrophysical Journal*, **69**, 173, 1929.

spectra of B stars available at the Yerkes Observatory shows that a large number show broad and shallow lines for all elements. This type of line cannot, of course, be due to Stark effect. C. T. Elvey[1] has obtained the contours of such "dish-shaped" lines for several stars and has called attention to the fact that they resemble the theoretical line contours as derived by Shajn and the writer[2] for rapidly rotating bodies. We consider this as good evidence that many B-type stars have a rapid axial rotation, and it is obvious that the Stark effect can be studied successfully only in objects that do not rotate or that have their axes so near to the line of sight as to cause but little rotational broadening of the lines. Because of the fact that elements of higher atomic number are much less affected by electric fields than the lighter elements, a search was made for stars showing narrow lines of ionized carbon (λ 4267), of ionized oxygen, of ionized magnesium (λ 4481), and of doubly ionized silicon.

A number of such stars were found, and the following discussion is based exclusively upon them. In many cases I have taken special spectrograms on Eastman Process plates, allowing the star to travel along the slit in order to obtain a widened spectrum. The results have been gratifying, and the information obtained from these plates has proved to be of considerable interest.

II. PRESSURE EFFECT IN HELIUM

In my paper on "The Stark Effect in Stellar Spectra"[3] I have pointed out that the members of various spectral series of any given element are affected in varying degree by electric fields. The members of the so-called "diffuse" series ($2P-mD$) are affected much more than the members of the "principal" ($2S-mP$) or of the "sharp" ($2P-mS$) series. This is in agreement with laboratory experience, and the very name "diffuse series" is due to the tendency of these lines to be hazy. However, they can be made quite sharp and narrow by reducing the pressure in the discharge tube.

An analysis of stellar spectra shows that the ($2P-mD$) lines of

[1] Unpublished.

[2] *Monthly Notices of the Royal Astronomical Society*, **89**, 222, 1929.

[3] *Astrophysical Journal*, **69**, 178, 1929.

He are invariably more hazy than the members of other series. This effect is very marked in many stars, and indeed it is somewhat rare to find a star where the difference in appearance between members of various *He* series is not very pronounced. The spectra of 88 γ Pegasi, of 85 ι Herculis, and of 17 ζ Cassiopeiae show the difference in haziness very clearly.

It appears also that the amount of haziness increases for the higher members of each series, so that the lines in the violet and ultra-violet parts of the spectrum are particularly well suited for comparison. The following groups are conveniently placed in the spectrum and show the effect clearly:

3926	$2P-8D$	very hazy
3965	$2S-4P$	very narrow
4009	$2P-7D$	hazy
4026	$2p^3-5d^3$	hazy
4121	$2p^3-5s^3$	narrow
4144	$2P-6D$	hazy
4169	$2P-6S$	narrow
4388	$2P-5D$	hazy
4438	$2P-5S$	narrow
4472	$2p^3-4d^3$	hazy

This effect provides a good proof that the widening of the *He* lines is actually due to molecular electric fields and not to Doppler effect. Indeed, the latter affects all atoms of a given element to the same extent and is consequently a function of temperature and atomic weight, but not of spectral series relationship.

A closer inspection of a large number of stellar spectra has revealed the fact that the difference in haziness between $(2P-mD)$ lines and members of other series is not constant. A number of stars were found where this difference, which we shall designate as ΔH, is small, and these were invariably stars with pronounced c-characteristic in their spectra. A typical example is 67 Ophiuchi, which is marked in the *Henry Draper Catalogue* as having narrow lines. An inspection of our plates of this star shows that the hydrogen lines are unusually free from the wide wings that are so characteristic of stars of early type in general. Furthermore, this star has either no component, or a very weak one, at λ 4470, although the ordinary *He* line, λ 4472, is strong and narrow. We conclude

that all characteristics of the Stark effect are absent; there are no forbidden lines of *He*, the *H* lines are narrow, and the various *He* series show little difference in haziness. A similar behavior is recorded for 21ϵ Canis Majoris and for 44ζ Persei and probably also for a small number of other stars.

There can be no doubt that we are dealing here with an effect of absolute magnitude: The luminous stars have rare atmospheres in which the Stark effect is greatly reduced while in the more compact dwarfs the electric fields are much stronger, on the average, because of the closer packing of the atoms and electrons. It seems probable that the variable quantity ΔH, defined as the difference in haziness of a (2P−mD) line and a (2P−mS) line, would give useful results in the determination of spectroscopic parallaxes.

III. IDENTIFICATION OF THE LINE AT λ 4470

In my paper on the Stark effect I tentatively identified the unknown absorption line at λ 4470.046 (on Rowland's scale) = 4469.88 I.A. with the strongest forbidden line of *He* which makes its appearance in the presence of electric fields. This identification can now be put beyond any reasonable doubt. In the first place, we have several determinations of its wave-length in stellar spectra:

Struve	4469.88 I.A.
Elvey	4469.86
Elvey, revised	4469.77
Mean	4469.84

The probable error of my measurements was ± 0.035 A. It is probable that the mean value given above will not be in error by more than about 0.03 A.

The wave-length of the forbidden line as it appears in the laboratory can be determined with the help of data contained in recent articles by J. S. Foster[1] and by H. Nyquist.[2] Their results agree well. They are given below for zero electric field:

Foster	4469.87 I.A.
Nyquist	4469.96
Mean	4469.92

[1] *Proceedings of the Royal Society*, A, **114**, 57, 1927.

[2] *Physical Review*, **10**, 235, 1917.

The laboratory value exceeds the stellar wave-length by an amount slightly greater than the probable error given above, but the difference is in agreement with the direction in which the line is displaced by the electric field. The observed difference is 0.08 A. Nyquist gives for the displacement of this line in an electric field the value -0.52×10^{-4} A$\times$cm/volt. If we were dealing with a constant field its intensity would be given by the ratio of these two quantities. In our case this ratio will merely indicate a certain average field-intensity between neighboring atoms. The actual value thus found is 1.5×10^{3} volt/cm, in good agreement with my earlier suggestion[1] that in such stars the average field should lie between 10^{3} and 10^{4} volt/cm.

While the evidence presented in favor of the identification with forbidden *He* would appear to be satisfactory, it must be shown that no other element will also fit the case. A careful search has been made among all the elements whose spectra are likely to be present in stars of such early spectral type. There appears to be only one possibility: Ionized oxygen shows a line at 4469.32. This line was measured by A. Fowler[2] and by J. Lunt,[3] and the two wave-lengths are:

Fowler	4469.32 I.A.
Lunt	4469.31
Mean	4469.315

The precision of the mean is probably such that it cannot be in error by more than 0.01 A. This in itself precludes its being indentical with the stellar line. But there are other grounds for discarding

[1] This result is consistent with the pressure as given by the theory of ionization. Using the method of Russell and Stewart (*Astrophysical Journal*, **59**, 203, 1924) we find the following expression for the pressure of the free electrons, in atmospheres:

$$p'=(3.5\times10^{-10})\cdot F^{\frac{3}{2}}\cdot T$$

where F is the average intensity of the field, expressed in c.g.s. electrostatic units, and T is the absolute temperature of the reversing layer. In our case the mean field is about 2000 volt/cm $=6.65$ electrostatic units. Substituting this in the above equation and assuming $T=20{,}000^{\circ}$ for a B-type star, we obtain approximately

$$p'=10^{-4}\text{ atmospheres}$$

This agrees well with the values given by R. H. Fowler and E. A. Milne and by H. N. Russell and others.

[2] *Proceedings of the Royal Society*, A, **110**, 476, 1926.

[3] *Annals of the Cape Observatory*, **10**, Part II, p. 26*B*, 1906.

this possibility. I have found that the line at 4470, as it appears in stellar spectra, is entirely uncorrelated with the intensities of the more prominent elements except *He*. It may be strong in stars showing either no oxygen at all or very weak oxygen, and it may be absent in stars where the oxygen lines are prominent. On the other hand, there is a distinct correlation with the intensity of the ordinary *He* line 4472. The forbidden line is never seen if the ordinary *He* line is weak or absent. It is usually strong in stars where 4472 is strong, and fails to appear only in such stars where the *H* lines are narrow and where the difference in haziness ΔH (discussed in the preceding section) is small.

IV. OTHER FORBIDDEN HELIUM LINES

Mr. Elvey[1] has shown that there is good reason to believe that the unsymmetrical appearance of the lines at 4388 and at 4922 is due to forbidden *He* components blended with the main lines. This is what might have been expected from the behavior of these lines in the laboratory. The forbidden lines are fainter and probably more complex than in 4472, so that their separate appearance cannot be expected. It seemed worth while to look for a violet component in the next member, following 4472, of the diffuse series of the triplet system. The line in question is 4026. In the laboratory this line divides into several components that shift from zero toward the red. There should also be a forbidden component at a somewhat smaller distance from the main line than in the case of 4472. Such a line has been measured in 88 γ Pegasi and especially in 85 ι Herculis. Its stellar wave-length (which is still very uncertain) is 4025.24 I.A. The adopted laboratory wave-length of the strong diffuse triplet line is 4026.189 I.A. The measurement of the forbidden component is somewhat complicated by the proximity of the ordinary (2P − 7S) *He* line at 4023.99 I.A.

In order to investigate other forbidden *He* lines we return to the diagram of Merton,[2] which gives a rough idea as to the expected intensities of the various lines. Unfortunately, the evidence concerning intensities is somewhat uncertain in view of the well-recognized

[1] *Astrophysical Journal*, **69**, 237, 1929.

[2] *Ibid.*, p. 190, 1929; *Proceedings of the Royal Society*, A, **95**, 30, 1919.

fact that laboratory intensities do not always correspond exactly to the observed stellar intensities.

We find that there is a bare chance that the two forbidden lines at 4519 and at 4047 may be visible. In approximately the correct position there are two unidentified lines recorded by F. E. Baxandall[1] in the spectrum of γ Orionis—a star which has a strong component at 4470 and should consequently be expected to show a strong development of the other forbidden lines. Their wave-lengths are given as 4518.6 and 4048.5 A (Rowland). I have suspected these lines in several other stars, but I have not been able to decide definitely whether they are real.[2] There is one other forbidden line at 4908 that might be present. At first it seemed that such a line actually existed, but measurement showed that the wave-length is 4906.644 I.A. and that it is probably identical with the O^+ line at 4906.88 I.A.

While our evidence concerning other forbidden lines is somewhat scanty, this is what we should have expected from the relative intensities of these lines. I think that there is no reason to doubt the correctness of our identification.

V. PRESSURE SHIFTS

In view of the fact that nearly all *He* lines widen unsymmetrically in the mol-electric Stark effect, this element is particularly well suited to a study of the displacements in wave-lengths caused by pressure. In my former paper I gave some evidence on this point gathered from S. Albrecht's rediscussion of a large number of measurements by E. B. Frost and W. S. Adams on *He* stars. These measurements were all made on three-prism plates and refer therefore to the region in the spectrum between λ 4400 and λ 4750. In

[1] Solar Physics Committee, *Comparison of the Spectra of Rigelian, Crucian and Alnitamian Stars*, p. 1, 1914.

[2] At my request Mr. Baxandall has looked up his plates of γ Orionis and has re-examined them. He finds that the two lines near 4047 and 4519 are probably real. The following is quoted from his letter: "I have little doubt that there is in each case a weak line very near the positions 4047 and 4519, which you give." Their wave-lengths, which may be in error by a few tenths of an A, are 4048.0 and 4519.2. He adds: ". . . . I would not be too dogmatic about the identification of such faint and difficult lines with the forbidden lines of *He*. . . . . I know of no other satisfactory origin for the lines discussed "

the present study I have made a number of measurements on plates, focused in the region of λ 4000, with a dispersion of one prism. The plates used were all of good quality, Eastman Process emulsion, and a wide star-window having been used. Some of the plates were remeasured by Mr. P. C. Keenan (these are marked by an asterisk in Table I). Table I shows the wave-lengths derived after correcting for the radial velocity of the star. The first line gives the adopted laboratory wave-length. For the triplets this value represents the probable blend between the two stronger components, the third line being generally too weak to affect the wave-length. It should be noted that in the determination of wave-lengths the singlets are more reliable than the triplets partly since the relative intensities of the blended multiplet components are not known and partly since it is a well-established fact that a blend will give wave-lengths dependent upon the density of the plate.[1] The second line shows the series relationship for each line. The third gives the amount in A-units by which the line is shifted in an electric field of 1 volt/cm. The relationship between shift and field strength can be taken as a straight line. The results of the measurements are arranged in two groups. The first group contains the stars 85 ι Herculis, 88 γ Pegasi, and 17 ζ Cassiopeiae for which the Stark effect was found to be appreciable (from the difference in haziness, ΔH, and from the intensity of the forbidden component at 4470, as well as from the width of the H wings). The second group contains three measurements of two plates of 67 Ophiuchi which we had suspected before to be almost free of Stark effect.

A comparison of various lines of the first group shows that the line 3965, which is known to broaden toward the violet, actually gives a much smaller wave-length than that adopted in the laboratory. On the other hand, the line 4169, which broadens toward the red, gives a positive excess in wave-length. The diffuse lines 4009 and 4026 give very nearly the same wave-length as that adopted in the laboratory, but here the actual values of the broadening factors in the second line are uncertain, owing to the presence of

[1] S. Albrecht, *Astrophysical Journal*, **57**, 61, 1923. It should be noted that neither the *He* lines nor the C^+ 4267 or the Mg^+ 4481 lines are divided on our plates. The multiplet components are separated in all these lines by approximately 0.2 A, which is not resolved with our dispersion.

many forbidden components, and these have, consequently, been placed in parentheses. In any case the great negative difference (Measured—Adopted) for 3965 and the positive difference for 4169 are very suggestive. The total displacement is nearly 0.1 A in each case, which is difficult to explain by error of measurement. There remains the possibility that our measurements are affected by systematic errors. I have investigated this possibility by measuring plates of α Lyrae and of α Cygni taken under the same conditions,

TABLE I

WAVE-LENGTHS OF HELIUM LINES

	Line	3964.727	4009.267	4026.189	4120.812	4143.759	4168.972
	Series relation	$2S-4P$	$2S-7D$	$2p^3-5d^3$	$2p^3-5s^3$	$2P-6D$	$2P-6S$
	$\Delta\lambda$	−0.22	(+1.0)	(+1.0)	−0.03	(+1.0)	$+0.43\times10^{-4}$
Strong Stark Effect	85 ι Her.	3964.675	4009.315	4026.199	4120.871	4143.703	4169.110
	85 ι Her.*	.688	.280	.190	.837	.722	.151
	88 γ Peg.	.579	.201	.122	.924	.912	.060
	88 γ Peg.	.605	.271	.158	.862	.834	.091
	88 γ Peg.	.633	.293	.188	.810	.933	....
	17 ζ Cas.	.617	.255	.147	.833	.812	.064
	Mean	.632	.269	.167	.856	.819	.095
Weak Stark Effect	67 Oph.	.734	.115	.077	.812	.716	8.918
	67 Oph.	.674	.176	.202	.847	.693	8.959
	67 Oph.*	.647	.155	.071	.852	.823	9.033
	Mean	.685	.149	.117	.837	.744	8.970

NOTE.—The values in parentheses in the third line refer to the main lines not blended with forbidden components.

but there is no indication of such an error. I cannot conceive at the present time how an error of the kind described could have affected our results, and in any case an error varying with the wave-length would also have affected λ 4009 and λ 4026, while in reality these two lines agree well with the adopted values.

The wave-lengths of the comparison lines cannot be responsible for the result since they agree well among each other and since the same lines were not always used.

Nevertheless, I feel that the possibility of a systematic error is not altogether excluded, and it is consequently safer to compare each line in the two groups with and without Stark effect. Here I

think we are perfectly safe that no systematic errors have crept in, since the plates were taken under similar conditions and since the methods of measurement and reduction were practically identical for the two groups. This comparison is very interesting. It shows that the line 3965 is less displaced in 67 Ophiuchi than in the first group. The wave-lengths of λ 4009 and λ 4026 are much more negative, suggesting that they were actually displaced toward the red in the first group. In agreement with the very small displacement suffered by this line in the electric field, λ 4121 is affected very little. On the other hand, λ 4169 gives a smaller value in 67 Ophiuchi, also in accord with the theoretical requirement that this line should widen toward the red in the electric field.

If we accept the results for the first group giving a relative shift of 0.2 A between 4169 and 3965, we find that this value would correspond to a field of about 3×10^3 volt/cm.

Returning to the wave-lengths collected by Albrecht,[1] we find that here, too, the star 67 Ophiuchi gives a smaller wave-length for 4388 than do the other stars which are more affected by Stark broadening. But in the case of the line 4472 this is not true. Whether the failure is due to the triplet structure of this line, or whether the amount of red shift, which is theoretically three times less than for the line 4388, is insufficient to bring out the effect more clearly, cannot be decided at this stage.

I regard these results on wave-lengths as quite preliminary. In my opinion they are much less reliable than the evidence afforded by the difference in haziness and by the intensity of the forbidden lines, but as far as they go they seem to corroborate the earlier results.

VI. THE N^+ LINES

Among the lines of other elements those due to N^+ are known to be very wide if produced under atmospheric pressure and sharp only under conditions of very low vacuum. In a study of N^+ in stellar spectra Baxandall[2] found that those lines which appear hazy in the laboratory have also a tendency to be hazy in the stars. I

[1] *Ibid.*, **67**, 305, 1928.

[2] Solar Physics Committee, *Researches on the Chemical Origin of Various Lines in Solar and Stellar Spectra*, p. 52, 1910.

have checked this result on a number of good spectrograms. Unfortunately, the N^+ lines in question are always faint and in many stars they are barely visible. Nevertheless there is good reason to believe that the two lines 4237 and 4242 appear hazy in stars with strong 4470 and with other indications of Stark effect. The same lines, on the other hand, look rather sharp and well defined in such stars as ϵ Can. Maj. or ζ Persei, which have already been mentioned as being devoid of any appreciable evidence of Stark effect. For practical purposes, such as the determination of absolute magnitudes, the N^+ lines are unsuitable on account of their extreme faintness. The only N^+ line which occurs with fair strength in stellar spectra is 3995. This line is always rather sharp, and it is not mentioned by A. Fowler and L. J. Freeman[1] as being particularly sensitive to pressure widening.[2]

VII. LINES OF OTHER ELEMENTS

Most of the other elements have rather sharp lines and there is no evidence that any of them show effects of pressure. There is some suspicion that the O^+ lines are not all alike in sharpness, but I have not been able so far to get reliable data on this point. The Si^{++} lines ($\lambda\lambda$ 4552, 4567, 4574), which are occasionally quite strong, are very sharp in all the stars used for this work. They seem not to be sensitive to pressure, and their widening, wherever it occurs, must be due chiefly to rotation. The C^+ line at 4267 is quite unusually sharp, and has served as a standard for testing the absence of appreciable rotation. The Mg^+ line 4481 has caused some trouble in a few instances. In the majority of stars investigated it appeared as a very sharp and narrow line, but in a few stars, particularly those of earliest spectral types, it appeared suspiciously wide. I am not quite certain whether this widening is real. If it is it may be due to the fact that λ 4481 is somewhat sensitive to pressure widening in the laboratory.[3]

[1] *Proceedings of the Royal Society*, A, **114**, 662, 1927.

[2] Mr. Baxandall has very kindly re-examined his plates and has verified his earlier results: "I would quote lines 4035.1, 4041.5, 4236.9, 4241.9, and 4530.1 as being the best representatives of the nebulous lines, as against 3995.3, 4447.2, and 4630.7 representing the type of line which is sharply defined, both in stellar and laboratory spectra."

[3] A. Fowler, *Report on Series in Line Spectra*, p. 118, 1922.

VIII. INTENSITIES OF HELIUM LINES

In the course of this work it was noticed that in each spectral series the *He* lines display a tendency to increase in intensity toward the members of shorter wave-length. This property is well known in the case of *H*, and it has been recorded in a few individual cases for *He*. The effect turns out to be rather universal, although it is

TABLE II

ESTIMATES OF LINE INTENSITIES IN 85 ι HERCULIS

DIFFUSE TRIPLETS		
4472	$2p^3-4d^3$	15
4026	$2p^3-5d^3$	15
SHARP TRIPLETS		
4713	$2p^3-4s^3$	4
4121	$2p^3-5s^3$	6
DIFFUSE SINGLETS		
4922	2P−4D	10
4388	2P−5D	13
4144	2P−6D	11
4009	2P−7D	8
3926	2P−8D	4
PRINCIPAL SINGLETS		
3965	2S−4P	8
SHARP SINGLETS		
4438	2P−5S	3
4169	2P−6S	4
4024	2P−7S	2
3935	2P−8S	1

not constant. The intensity within each series increases at first, and then decreases rapidly toward the series limit. Visual estimates for 85 ι Herculis are given in Table II.

In a few stars the decrease in intensity seems to start earlier than in others, but in all cases thus far examined the *He* lines in the region around 3900 to 4000 are much stronger than one would have expected from the laboratory intensities.

In this connection I wish to correct an error in my paper on the

detached lines of calcium in stellar spectra.[1] In that work I used a spectral line near *CaK* as a control. This line I identified as Si^{++} 3924 / ~~4024~~. It is clear that I was dealing chiefly with the *He* line ~~4026~~, / 3926 which is probably blended on plates of small dispersion with the Si^{++} line. This makes no difference in the results, and the distribution of the intensities as observed for the control line agrees with the expected behavior of the blend as well as with the Si^{++} line alone. The error obviously arose from the fact that the *He* line was thought to be too faint to be present. In reality it is fairly strong, owing to the peculiarity noted in this section.

An explanation for the unusual behavior of the *He* intensities is still lacking. For hydrogen H. N. Russell has mentioned that the Stark effect would cause some such displacement in the intensities,[2] but whether this can account for the whole effect is not certain. A similar effect has been observed by Merton and Nicholson in the laboratory,[3] but its precise physical interpretation is not known. It is possible that it may be ultimately traced to the pressure.

The peculiar differences in the intensities of triplets and singlets has recently been pointed out.[4] This has been confirmed by Elvey's contours. It is quite possible that here, too, we are dealing with an effect of pressure, but, as I have shown,[3] the simple explanation of certain laboratory experiments by Ornstein, Burger, and Kapuscinski[5] cannot be applied to the stars.

IX. MAXIMA OF ABSORPTION LINES

A large amount of work has recently been done in connection with the exact determination of the maxima of certain lines in the spectral sequence. The interpretation of these results according to the theory of ionization is particularly important in many respects, and it may therefore not be out of place to call attention here to a difficulty that arises in the case of lines which are subject to strong Stark broadening.

According to the theoretical work of J. Q. Stewart and of

[1] *Astrophysical Journal*, **67**, 356, 1928. [2] *Ibid.*, **70**, 64, 1929.

[3] *Philosophical Transactions of the Royal Society*, A, **217**, 237, 1926; *ibid.*, A, **220**, 137, 1918.

[4] *Nature*, **122**, 994, 1928. [5] *Zeitschrift für Physik*, **51**, 34, 1928.

A. Unsöld, the width of a line is dependent upon the abundance of atoms capable of absorbing the given frequency. Russell and Miss Moore have shown that such a mechanism, as contemplated by Stewart and Unsöld, is actually responsible for the winged lines in the solar spectrum, and Miss Payne has obtained good results for the Ca^+ lines in almost all spectral types.

According to Stewart, the width of a line is given by

$$W=(8.15\times 10^{-13})\cdot\lambda\cdot\sqrt{\frac{N\cdot D}{k}}\,,$$

where λ is the wave-length, N is the number of active atoms per cubic centimeter, D is the length of the absorbing column in centimeters, and k is the contrast factor. The number of atoms per cubic centimeter is proportional to the fractional concentration of atoms in the state appropriate for the absorption of the given line, and this is given by the theory of ionization. Accordingly, we expect W to be at a maximum when the concentration is greatest. As the observed central intensity also depends upon the concentration, the line width and central intensity, as well as the total absorbed energy, should all be at maximum at the same point of the spectral sequence.

This simple theory has recently been modified by E. A. Milne. He has called attention to the fact that different portions of the line contour are originated at different effective levels of the reversing layer and that consequently the wings are produced in denser and presumably slightly hotter parts of the atmosphere than the central core of the line; in consequence the maximum must be considered separately for each optical depth. As a first approximation Milne predicted that the H lines would reach their maximum in the wings at an earlier spectral type than the core of the line. This prediction is not fulfilled by the observations of Miss Payne and Miss Williams[1]. On the other hand, the unpublished results of Elvey indicate that for the He lines the displacement is in the direction predicted by Milne. Finally, for the Ca^+ lines, there is very little, if any displacement, according to Misses Payne and Williams.

[1] *Monthly Notices of the Royal Astronomical Society*, **89**, 526, 1929.

It seems clear that a line in which the width is largely dominated by mol-electric Stark effect must be considered separately, and since this is particularly true of *H* and to a less extent also of *He*, it becomes of considerable importance in the interpretation of the results on maxima.

We shall assume that the observed width of the line is composed of two factors. The first is due to the abundance of the atoms in the proper state, and will act in accordance with the formula of Stewart. The second is caused by the Stark effect and is thus dependent upon the pressure:

$$W = W_1 + W_2 ,$$

where

$$W_2 = \text{Const. } F = \text{Const. } \frac{e}{x^2} ,$$

and e = electronic charge, x = average distance between a charge and a radiating atom.

Further:[1]

$$x = c \cdot N^{-\frac{1}{3}} \quad \text{and} \quad N \propto \frac{p'}{T} .$$

Consequently,

$$W_2 = \text{Const. } e \cdot \left(\frac{p'}{T}\right)^{\frac{2}{3}} .$$

We know very little concerning the distribution of pressures in stellar atmospheres. If the pressure is independent of spectral type, the additional term in the line width, W_2, causes no change in the position of the maximum. However, there is no reason to believe that the pressure is constant all along the spectral sequence. If it is variable, W_2 may cause a marked displacement in the maximum of the line widths against the position predicted by the theory of ionization.

The formula developed by Fowler and Milne for the concentration of atoms of a given state is rather complicated. To simplify the computations we shall test the effect for a curve of a simpler type. Suppose that W_1 is given by a parabolic expression:

$$W_1 = A + B \cdot S^2 , \tag{1}$$

[1] *Astrophysical Journal*, **69**, 185, 1929.

where S is the spectral type in any convenient unit. The maximum, in the absence of W_2, is obtained by differentiating:

$$\frac{dW}{dS}=\frac{dW_1}{dS}=2BS=0\ ;\qquad S=0\ .$$

If the second term, W_2, is constant our result is unaltered, but if it is a function of S the result is affected. Suppose, for example, that

$$W_2=C\cdot S+D\ . \tag{2}$$

Then the maximum is obtained by differentiating (W_1+W_2):

$$\frac{dW}{dS}=2BS+C=0\ ;\qquad S=-\frac{C}{2B}\ . \tag{3}$$

The position of maximum is shifted from its normal position.

There is at present not enough evidence to evaluate the actual function connecting W_2 with S. But to illustrate the order of magnitude of the possible shift we shall carry through an approximate computation.

In the first place we evaluate the constants A and B in formula (1). From the diagram by Fowler and Milne, for hydrogen,[1] we find that the parabolic equation satisfactorily represents the upper part of the curve, if we assume 0.1 of a spectral class as unit of S. Maximum takes place, theoretically, at about A0. At A4 the concentration is reduced about five times. Remembering that $W_1\sim\sqrt{N}$, we find for our constants

$$A=+2.2\,,$$
$$B=-0.075\,,$$

so that

$$W_1=2.2-0.075\cdot S^2$$

We assume here that for A4 the value of W_1 is unity, and for A0, $W_1=\sqrt{5}$. In order to obtain at least a rough guess of how W_2 may depend upon the spectral type, we write, according to Milne.[2]

$$p'\sim\frac{g^{\frac{1}{3}}}{T^2}\,,$$

[1] R. H. Fowler, *Statistical Mechanics*, p. 380, 1929.

[2] *Monthly Notices of the Royal Astronomical Society*, **85**, 782, 1925.

where g is the surface gravity and T is the temperature. Therefore,

omitting of T here

$$W_2 \propto \frac{g^{\frac{1}{3}}}{T^{\frac{4}{3}}}.$$

The surface gravity, g, is a function of mass and radius of the star:

$$g = \frac{\gamma \cdot \mu}{R^2}.$$

so that

$$W_2 \propto \frac{\mu^{\frac{1}{3}}}{R^{\frac{2}{3}} T^{\frac{4}{3}}}.$$

We now wish to express W_2 as a function of the absolute bolometric magnitude M. From J. H. Jeans[1] we take:

$$\log R = -0.2\, M - 2 \log T + 8.53.$$

Eddington's mass-luminosity relationship gives μ as a function of M. In the form of Jeans:[2]

$$\log \mu = -0.133\, M + 0.645.$$

Substituting into W_2 we have, by taking logarithms:

$$\log W_2 = 0.1\, M + \text{Const.}$$

Therefore, the more luminous the star the narrower should be its lines. From statistical data on absolute magnitudes we find that M changes by about $1^{\mathrm{M}}.5$ between A0 and A4. Consequently, W_2 changes 1.4 times. Although the relationship between W_2 and S is not strictly linear, we shall assume it to be so in order to obtain a rough orientation as to the size of the effect. Consequently, we find that a change of W_2 over a factor of 1.4 in 4 units of S corresponds to a value of

$$C = -0.1,$$

if at A4, $W_2 = 1$. We consequently assume that at this point $W_1 = W_2$. D is not important as it does not appear in the expression for the maximum (3). Substituting B and C into (3) we obtain for the position of maximum line width

$$S_{\text{max.}} = +1 \text{ unit.}$$

[1] *Astronomy and Cosmogony*, p. 47, 1928. [2] *Ibid.*, p. 126.

The maximum line width is shifted by 0.1 spectral class toward later types.

In a similar way it can be shown that the maximum of the central intensities will be displaced in the opposite direction by an amount of about the same order of magnitude. Consequently, the total effect may cause a displacement in the maxima of the core and of the wings amounting to 0.2 spectral class. This is true for the case where W_1 and W_2 are about equally important. In reality, a great many lines are practically free from Stark effect, and in these W_2 will be insignificant. On the other hand, it is possible that for H the contribution of W_2 to the total line width is even greater than in our hypothetical example.

X. CONCLUSIONS

Experimental data on the Stark effect have enabled us to make the following predictions for stellar absorption lines: (1) Their width should increase toward the higher members of each series. (2) The members of the "diffuse" series should appear hazier than the members of other series. (3) There should be present some of the forbidden lines, whose appearance is stimulated by electric fields. (4) Many lines should broaden unsymmetrically, causing pressure shifts in the observed stellar wave-lengths.

In the preceding sections of this paper we have seen that the last three points are in perfect agreement with the observations as far as the *He* lines are concerned. The first predication caused some trouble in my discussion of the *H* lines.[1] The effect is more pronounced in *He*, where the width and haziness of the diffuse series rapidly increase toward the higher members. This is clearly shown by the diffuse series of the singlet system where we observe simultaneously five members. Thus λ 4922 is usually rather narrow, while λ 3926 is so diffuse that the edges can hardly be measured. However, it seems that in *He* as well as in *H* the increase in width is not as rapid as one would be led to suspect from theoretical considerations. This discrepancy is not serious enough to throw doubt upon the evidence of the other three predictions, but a satisfactory explanation must be found for it. I have suggested several such

[1] *Astrophysical Journal*, **69**, 193, 1929.

explanations in my first paper.[1] But the most effective cause probably lies in the mechanism by which a line is produced in a stellar atmosphere. The theoretical calculations apply only to a gas of uniform density. The reversing layer of a star is however highly heterogeneous with respect to density. The lower strata will act according to the theoretical prediction, and should produce rapidly widening wings. But the higher strata will superimpose over each such wing a narrow line devoid of pressure widening, and the result will probably be a tendency in the measured line widths to increase slower than anticipated.[2]

The variability of the quantity ΔH and its probable correlation with pressure provides a new method for the determination of spectroscopic parallaxes. It appears that stars known to be giants show small values of ΔH. At the same time the forbidden line λ 4470 is either absent or very faint, while the H lines do not show the wide wings which are so characteristic of the stars with more pronounced ΔH and with strong λ 4470. I hope to develop these three criteria and to calibrate them empirically from known absolute magnitudes. However, it should be remembered that the method is applicable to only a comparatively small group of stars where axial rotation is not present in any large degree.

It is a pleasure to acknowledge the assistance received from Dr. A. Pogo in the earlier phases of this work. Mr. P. C. Keenan has kindly measured some of the plates used in Table I. My thanks are also due to Mr. C. T. Elvey for advance information concerning his contours of the *He* lines in stellar spectra, and to Mr. F. E. Baxandall for the verification of his earlier results on certain lines in B-type stars.

YERKES OBSERVATORY
August 6, 1929

[1] *Ibid.*

[2] It is the opinion of Dr. A. Unsöld that this difficulty can be overcome by theoretical methods.

THE ELECTRIC-FURNACE SPECTRUM OF HAFNIUM[1]

By ARTHUR S. KING

ABSTRACT

The spectrum of metallic hafnium has been examined from λ 2640 to λ 6500 in the carbon-tube *furnace* and compared with the spectra of the *arc* and *spark*. A list of *338 lines* includes the stronger neutral and ionized lines in this range. The usual *temperature classification* is given for the lines of the *neutral spectrum*, based on their relative intensities at 2900° and 2600° C. The lines which, judged by their relative intensities in arc and spark, belong to the *ionized atom* are *absent* from the furnace spectrum. The ionized lines are selected on the basis of their behavior in the three sources.

Attention is called to a *band spectrum* appearing in the arc in air and presumably due to the *oxide*. *Wave-lengths* are given for the *first head* in each of *nine groups* between λ 3200 and λ 5700.

The discovery of the element hafnium, by means of its Röntgen-ray spectrum, was announced by D. Coster and G. v. Hevesy[2] in January, 1923. During the same year, measurements of wave-lengths from λ 2254 to λ 7241 were published by H. N. Hansen and S. Werner.[3] A recent paper by W. F. Meggers[4] gave extended data on this spectrum, consisting of wave-lengths of nearly 1500 lines between λ 2155.72 and λ 9250.27, with their intensities in arc and spark. Regularities among the singly ionized lines were also found by Meggers and Scribner.[5] It was obviously desirable that the next investigation be made with the electric furnace in order to show the relative response of the lines to various excitations, and to render more definite the division of the spectrum into lines of the neutral and of the ionized atom.

It has been recognized since its discovery that hafnium is likely to be associated with zirconium to the extent of a fraction of 1 per cent, but the separation is difficult, and hafnium in any sort of purified form is still extremely scarce. The writer is greatly indebted to Dr. G. Holst, director of the Research Laboratory of the Philips

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 384.

[2] *Nature*, **111**, 79, 1923.

[3] *Ibid.*, p. 322, 1923; **112**, 618, 900, 1923.

[4] *Bureau of Standards Journal of Research*, **1**, 151, 1928.

[5] *Journal of the Optical Society of America*, **17**, 83, 1928.

Lamp Works in Eindhoven, Holland, for his kindness in furnishing a small rod of highly purified hafnium for this work. No troublesome impurities were present in this sample, even the prominent lines of zirconium being quite faint.

EXPERIMENTAL METHOD

In these experiments, the furnace, arc, and spark spectra of hafnium were photographed within the range λ 2640 to λ 6500. A comparison of my results with those of Meggers shows that the neutral lines outside this interval are so faint that it is doubtful if the furnace can give useful data farther toward either the ultra-violet or the infra-red. The spectrograms from the ultra-violet to λ 4900 were made in the second order of the 15-foot concave grating (scale 1.86 A per millimeter), and beyond this in the first order.

In charging the furnace a thin graphite combustion boat containing fragments of hafnium was placed in the resistor tube. The metal was changed into a dark powder by the operation of the furnace, and the boat with its contents was used in successive tubes during the experiments. The melting point of hafnium has been measured in the Philips Laboratory as near 2400° C., and I found a temperature of 2600° necessary to bring out any considerable number of lines. Temperatures of 2900°–3000° were required to do justice to the neutral spectrum. Although at the highest of these temperatures some of the weaker arc lines were still absent from the furnace spectrum, 2600° and 2900° were taken as the two stages for the classification of furnace lines. Because of the high temperatures required, the Swan bands of carbon were always strong in the furnace spectrum, and occasionally masked the hafnium lines which would be expected to appear.

The electrodes used for the arc and spark were silver rods 6 mm in diameter, the lower having in its end a small piece of hafnium. A comparison of the arc of 4 amp at 220 volts with a very disruptive transformer spark served to distinguish clearly between the neutral and ionized lines, the former being very faint in the spark spectrum. As the lines enhanced in the spark, even when of considerable strength in the arc, were quite absent from the furnace spectrum, it was not possible to employ the usual test for enhanced lines of

quenching them by a mixture of the substance in the furnace with one of lower ionization potential.[1] The selection of enhanced lines must therefore rest on their absence from the furnace and their strengthening in the spark; in nearly all cases, however, this effect was so definite that there is no likelihood of error in distinguishing them.

EXPLANATION OF THE TABLE

The first column contains Meggers' wave-lengths on the international system and also a few measurements by the writer of lines probably unresolved on Meggers' plates or disturbed by foreign lines. The neutral lines listed are those present in the furnace spectrum, or, if absent, are those of sufficient strength in the arc to make their absence at the furnace temperatures significant. Enhanced lines are included only when of considerable strength in the arc. Estimated intensities for the arc spectrum are in the second column. It must be remembered that the relative intensities of neutral and ionized lines, as groups, vary greatly in different spectrograms according to the arc conditions. On this account consistent relative intensities can be given only between the lines of each group. The columns of furnace intensities for 2900° and 2600°, respectively, show the relative response of the neutral lines to temperature change. In the fifth column, the usual temperature classification is held to as closely as possible, although it must be taken into account that hafnium lines are in general faint in the furnace spectrum, and that only two temperature stages were used. The data do not justify placing any lines in class I. Those emitted with fair strength at 2600° are put in class II, while lines appearing only at high temperature are in class III or class IV according to their relative strength in furnace and arc. Neutral lines absent from the furnace are placed in class V, while all enhanced lines are classed as V E.

An asterisk adjacent to a wave-length indicates that an explanatory note concerning this line is given at the end of the table. Disturbance by the band spectrum of carbon, which frequently conceals hafnium lines in the furnace spectrum, is denoted by a dagger.

[1] *Mt. Wilson Contr.*, No. 233; *Astrophysical Journal*, 55, 380, 1922.

TABLE I

Temperature Classification for Hafnium

λ (I.A.) (Meggers)	Arc Int.	Furnace Intensity 2900°	Furnace Intensity 2600°	Class	λ (I.A.) (Meggers)	Arc Int.	Furnace Intensity 2900°	Furnace Intensity 2600°	Class
2641.42	100	.....	.....	V E	2866.38	150	4	3	II
2642.07	3	.....	.....	V	2876.33	20	.....	.....	V E
2642.76	5	.....	.....	V	2887.13	15	.....	.....	V
2647.31	60	.....	.....	V E	2887.54	15	.....	.....	V
2657.85	12	.....	.....	V E	2889.62	80	4	2	II
2661.89	20	.....	.....	V E	2898.26	125	6	4	II
2668.28	5	.....	.....	V	2898.71	20	.....	.....	V E
2683.36	30	.....	.....	V E	2904.42	80	4	1	III
2696.18	4	.....	.....	V	2904.76	80	4	2	II
2699.63	3	.....	.....	V	2909.91*	20	.....	.....	V E
2705.62	30	.....	.....	V	2916.49	300	6	4	II
2712.43*	40	.....	.....	V E	2918.58	80	3	1	III
2713.84	6	.....	.....	V	2919.59	100	.....	.....	V E
2718.57	8	.....	.....	V	2924.61*	8	1	.....	IV
2726.69	2	.....	.....	V	2928.98	2	.....	.....	V
2730.71	2	.....	.....	V	2929.63*	100	.....	.....	V E
2730.84	4	.....	.....	V	2929.90	40	.....	.....	V
2738.76	100	.....	.....	V E	2937.80	80	.....	.....	V E
2743.63	6	.....	.....	V	2940.77	200	10	8	II
2751.81*	30	.....	.....	V E	2944.71	15	2	.....	III
2758.76*	6	.....	.....	V	2950.68	125	8	4	II
2761.62	30	.....	.....	V	2954.21	100	5	2	II
2766.96	8	.....	.....	V	2958.02	40	2	.....	III
2773.01	8	.....	.....	V	2964.88	150	10	5	II
2773.37*	150	.....	.....	V E	2966.95	40	2	.....	III
2774.02	15	.....	.....	V E	2967.24	20	.....	.....	V E
2779.36	20	1	.....	IV	2968.83*	100	.....	.....	V E
2783.68	2	.....	.....	V	2973.38	6	.....	.....	V
2789.51	3	.....	.....	V E	2975.90	100	.....	.....	V E
2789.74*	6	.....	.....	V E	2979.28	8	.....	.....	V
2808.00	20	.....	.....	V E	2980.82	100	12	7	II
2813.86	20	.....	.....	V E	2982.73	8	.....	.....	V
2814.47	15	.....	.....	V E	2984.06	2	.....	.....	V
2817.68	5	.....	.....	V	3000.10	80	.....	.....	V E
2818.93	3	.....	.....	V	3005.56	100	3	.....	III
2819.75	8	.....	.....	V	3012.90	200	.....	.....	V E
2820.23	150	.....	.....	V E	3016.71*	30	4	1	III
2822.67	80	.....	.....	V E	3016.83*	80	12	6	II
2833.29	20	1	.....	IV	3016.93*	2	.....	.....	V E
2834.12	3	.....	.....	V	3018.32	80	15	8	II
2841.49	2	.....	.....	V	3020.54	100	10	5	II
2845.83	20	.....	.....	V	3024.61	4	.....	.....	V
2849.21	30	.....	.....	V E	3025.29	10	.....	.....	V E
2850.96	25	.....	.....	V	3031.17	100	.....	.....	V E
2851.21	30	.....	.....	V E	3049.30	2	.....	.....	V
2852.03	8	.....	.....	V E	3050.76	60	3	2	II
2860.56	10	.....	.....	V	3054.52*	6	.....	.....	V E
2861.01	80	.....	.....	V E	3055.43	3	.....	.....	V E
2861.69	100	.....	.....	V E	3057.02	100	12	4	II

TABLE I—*Continued*

λ (I.A.) (Meggers)	Arc Int.	Furnace Intensity 2900°	Furnace Intensity 2600°	Class	λ (I.A.) (Meggers)	Arc Int.	Furnace Intensity 2900°	Furnace Intensity 2600°	Class
3063.77*...	6	—?	......	V ?	3220.66....	20	......	......	V E
3064.68....	8	......	......	V E	3227.87....	2	......	......	V
3067.41....	80	10	4	II	3230.07....	4	......	......	V
3069.21....	2	......	......	V	3236.77*...	8	......	......	V
3072.88....	300	20	15	II	3239.41....	3	......	......	V
3074.09....	10	1	......	IV	3243.36....	4	......	......	V
3074.78....	30	2	1	II	3247.66*...	20?	5	4	II
3080.63....	60	......	......	V E	3249.53*...	12	4?	2	II
3080.84....	80	8	3	II	3253.70....	100	......	......	V E
3091.37....	2	......	......	V	3254.86....	3	......	......	V
3092.25....	8	......	......	V E	3255.29*...	30	......	......	V E
3096.76....	15	2	1	II	3261.90....	3	......	......	V
3100.78*...	2	......	......	V	3262.47....	5	......	......	V
3101.38....	100	......	......	V E	3265.29....	4	......	......	V
3109.11....	200	......	......	V E	3267.01....	3	......	......	V
3110.88*...	20	......	......	V E	3267.17....	5	......	......	V
3119.97....	10	1	......	IV	3273.66....	8	......	......	V E
3126.65....	4	......	......	V	3291.04*...	8	......	......	V E ?
3128.75....	6	1	tr	III	3298.94....	2	......	......	V
3129.59....	4	1	tr	III	3306.12....	10	2	2	II
3131.81....	150	3	1	III	3309.20....	3	......	......	V
3134.72....	300	......	......	V E	3310.27....	25	4	3	II
3137.52....	8	......	......	V	3312.87....	100	12	12	II
3138.67*...	8	......	......	V	3316.18....	2	......	......	V
3139.67....	20	......	......	V E	3317.99*...	6	......	......	V E
3140.77....	10	......	......	V E	3328.21*...	4	......	......	V E
3145.33....	40	......	......	V E	3331.87....	2	......	......	V
3148.41....	15	1	......	IV	3332.73....	200	20	20	II
3151.63....	6	......	......	V	3352.06....	50	......	......	V E
3152.96....	2	......	......	V	3356.78....	2	......	......	V
3156.68....	50	5	3	II	3358.95*...	{2, 5}	1	tr	III
3159.83....	30	8	4	II					
3162.57*...	80	15	10	II	3360.06....	6	2	2	II
3162.62*...	15?	......	......	V E	3366.68....	3	......	......	V
3164.39*...	12	3	1	III	3378.93*...	{3, 5}	1	1	II
3165.73....	3	......	......	V					
3168.39....	20	1	......	IV	3384.70*...	3	......	......	V E
3172.94....	100	10	5	II	3386.21*...	5	......	......	V
3176.86....	75	......	......	V E	3389.83....	80	......	......	V E
3178.43....	2	......	......	V	3392.81....	4	......	......	V
3179.63....	2	......	......	V	3394.58*...	10	......	......	V E
3181.02....	15	2	1	II	3394.99....	3	......	......	V E
3181.13....	8	1	......	III	3397.26....	8	2	1	II
3189.63....	8	3	2	II	3397.60....	6	......	......	V
3193.53....	75	......	......	V E	3399.80....	300	......	......	V E
3194.20....	200	......	......	V E	3400.21....	8	5	3	II
3196.92....	10	1	tr	III	3402.51....	6	1	tr	III
3200.00....	12	......	......	V E	3407.13....	2	......	......	V
3206.11....	25	3	2	II	3407.76....	3	......	......	V E
3210.97....	8	1	......	III	3410.18....	6	......	......	V E
3217.30*...	20	......	......	V E	3412.38....	2	......	......	V

TABLE I—*Continued*

λ (I.A.) (Meggers)	Arc Int.	Furnace Intensity		Class	λ (I.A.) (Meggers)	Arc Int.	Furnace Intensity		Class
		2900°	2600°				2900°	2600°	
3417.35	6	3	2	II	3849.52	5			V E
3419.18	12	4	2	II	3858.30	20			V
3428.36*	7			V E	3872.54	3			V E
3438.43	5	1		IV	3880.81	10			V E
3440.88	2			V	3889.24	4			V
3441.84	3			V	3889.33	4			V
3448.30	2			V	3899.93	20	8	4	II
3452.31	4			V	3918.10*	20			V E
3462.65	4			V E	3923.91	4			V E
3467.56	6			V	3931.36	10			V
3472.38	100	15	10	II	3951.81	12	4		III
3478.93	8			V E	3968.01	3			V
3479.28*	40			V E	4032.27†	6	4?	?	III ?
3484.71	1			V	4062.84	10			V
3495.75	10			V E	4080.44	8			V E
3497.16	8			V	4083.36	5			V
3497.49	150	20	12	II	4093.17*	40			V E
3505.23	100			V E	4113.58	3			V E
3523.02	60	12	4	II	4127.78	6			V E
3535.54	60			V E	4145.76	6			V
3536.62†	30	8	?	II ?	4162.40	5			V E
3552.70	20			V E	4162.68	4			V
3561.65	100			V E	4174.33†	50	15	5?	II
3567.36†	6	?	?	V ?	4177.50	4			V E
3569.03	50			V E	4206.57	5			V E
3599.87	8			V	4209.71†	8	1?		III ?
3616.89	30	8	4	II	4228.08	6	4		III
3630.86	8	4	2	II	4232.46	5			V E
3644.35	40			V E	4249.33	2			V E
3649.09	10	4		III	4260.96	8			V
3661.05	2			V E	4263.41	10	6		III
3665.36	6			V E	4272.84	8			V E
3672.28	6			V	4294.77	30	8	4?	II ?
3675.74	10			V	4318.13†	6	3?	?	III ?
3682.25*	200	30	20	II	4320.67	3			V E
3696.52	5	3		III	4330.27	8			V
3699.73	5			V E	4336.65	10			V E
3701.15	6			V E	4350.50	8			V E
3717.80	40	15	5	II	4356.29	20	—?	—?	V ?
3719.27	40			V E	4367.89	2			V E
3733.78	15	4		III	4370.92*	10			V E
3746.80	10	1		III	4417.35	8			V E
3768.25†	4	3	—?	III ?	4417.90	10	4		III
3777.64	50	10	8	II	4422.75	3			V E
3785.46†	30	—?	?	IV ?	4438.02	20	3		III
3793.37	20			V E	4457.35*	12	3?		III
3800.39†*	10	?		IV ?	4461.17	15	3		III
3811.76	8	3		III	4540.92	20	3		III
3820.74†	50	5?	?	III ?	4565.94	20	2		III
3830.01	6			V	4598.80*	25	4?		III ?
3849.17	25			V	4598.92*	20			V

TABLE I—*Continued*

λ (I.A.) (Meggers)	Arc Int.	Furnace Intensity		Class	λ (I.A.) (Meggers)	Arc Int.	Furnace Intensity		Class
		2900°	2600°				2900°	2600°	
4620.87....	25	2	......	III	5294.87†...	12	?	......	III ?
4655.19†...	30	3?	?	III ?	5298.06....	5	......	......	V E
4664.13....	5	......	......	V E	5309.68....	2	......	......	V
4699.01....	4	......	......	V	5311.60....	8	......	......	V E
4782.75....	8	......	......	V	5354.73....	8	5	......	III
4800.50....	25	4	3	II	5373.86....	20	15	2	II
4859.23....	4	......	......	V	5389.33†...	3	?	......	V ?
4934.44....	2	......	......	V E	5452.92†...	10	2?	......	III ?
4948.94....	3	......	......	V	5550.60†...	50	?	?	III ?
4975.25....	25	4	......	III	5552.12†...	40	?	?	III ?
5018.20....	8	8	2	II	5613.27....	10	2	......	III
5040.82....	10	......	......	V E	5719.18....	40	5	......	III
5047.45†...	15	5?	?	III ?	5890.45†...	2	?	......	V ?
5079.63....	2	......	......	V E	5902.94....	15	8	......	III
5157.96†...	6	?	?	?	5974.28†...	4	?	......	V ?
5170.18....	5	......	......	V	6098.67....	3	......	......	V
5181.86....	40	30	4	II	6185.13†...	5	4?	......	III ?
5243.99....	15	2	......	III	6248.96....	2	......	......	V E
5275.04....	4	2	......	III	6386.23†...	6	6?	?	III ?

NOTES TO TABLE I

λ	
2712.43	Moderately enhanced. *Zr* II λ 2712.41 not same line.
2751.81	May be blend with *Hf* I.
2758.76	Measured by writer. *Zr* λ 2758.82 on red edge.
2773.37	Weaker *Hf* II line to red.
2789.74	Weaker *Hf* II line to red.
2909.91	Moderately enhanced.
2924.61	Not *Zr* II λ 2924.64.
2929.63	Moderately enhanced.
2968.83	Weaker *Hf* II line to red.
3016.71 3016.83	Measured by writer. Doublet given by Meggers as λ 3016 78.
3016.93	Moderately enhanced.
3054.52	Moderately enhanced.
3063.77	Blend *V* in furnace.
3100.78	Blend faint *Hf* II line.
3110.88	Measured by writer.
3138.67	Measured by writer. Coincides *Zr* II λ 3138.68.
3162.57 3162.62	Blended in arc.
3164.39	Measured by writer. *Zr* λ 3164.32 to violet.
3217.30	Moderately enhanced.
3236.77	Measured by writer.
3247.66	Measured by writer. Close to *Cu* line.

λ	
3249.53	Blend *Ti* in furnace.
3255.29	Moderately enhanced.
3291.04	Coincides with hazy line in spark. Meggers' intensities indicate *Hf* II
3317.99	Moderately enhanced.
3328.21	Moderately enhanced.
3358.95	Double.
3378.93	Double.
3384.70	Moderately enhanced.
3386.21	Measured by writer. Not *Ag*.
3394.58	Measured by writer. Moderately enhanced.
3428.36	Moderately enhanced.
3479.28	Moderately enhanced.
3682.25	*Ag* line to red.
3800.39	Faint *Hf* II line to red.
3918.10	Moderately enhanced.
4093.17	Moderately enhanced.
4370.92	Measured by writer.
4457.35	Blend *Ti* in furnace.
4598.80 4598.92	Measured by writer. Doublet given by Meggers as λ 4598.86. Disturbed by carbon.

DISCUSSION

Features of the line spectrum.—The results here given, if used in connection with those of Meggers, serve to show the excitation required for the production of individual hafnium lines. In the range common to the two investigations, the observations are in close agreement. Lines which I have assigned to the neutral spectrum, through the comparison of furnace, arc, and spark, are given by Meggers as much weaker in the spark than in the arc; while in Meggers' list the lines I have classed as ionized are stronger in the spark than in the arc, or, in some cases, nearly equal in the two sources. From this relation between the two sets of data, the more extended table of Meggers may be used to classify the lines which I have not listed. The strong line λ 4093.17, which Meggers considers as neutral, does not appear in the furnace, and, I believe, on account of its high spark intensity should be placed among the enhanced lines. It belongs to a group in the region of shorter waves shown by the relative intensities of Meggers, confirmed by my spectrograms, to be enhanced in the spark much less than the majority of the enhanced lines. An origin of low energy-level in the ionized atom is thus indicated. The fainter enhanced lines appearing in the arc, usually much strengthened in the spark, are omitted from my list, as are numerous neutral

lines too faint to be expected in the furnace spectrum. The furnace gives a number of lines, however, which are also among the weaker arc lines. This fact has justified the inclusion of neutral lines whose absence from the furnace and moderate intensity in the arc indicate that high excitation is required for their production. A large range of intensity is found among neutral lines in the arc, while in the furnace, probably on account of the small quantity of the substance and its high vaporization point, the lines are for the most part faint. The prominent hafnium lines, both neutral and ionized, are in the near ultra-violet. With some exceptions, such as λ5181.86, the production of neutral lines in the furnace becomes increasingly difficult toward greater wave-length.

Band spectrum.—A set of bands occurring in the hafnium arc spectrum may be noted. These show well-defined structure, in all cases degraded toward the red. The more prominent heads have been measured by Meggers. The following list gives the head of shortest wave-length in each group.

λ	λ	λ
3236.09	3840.03	4252.05
3327.82	3970.06	5074.71
3654.26	4101.16	5698.04

The writer measured λ 3236 and λ 4101, the first of which is absent from Meggers' list and the second disturbed by a foreign line on his plates. The heads given above are usually the strongest in their respective groups, but the λ 4101 system has a stronger head at λ 4118.91.

These bands are in all cases absent from the spectrum of the vacuum furnace. As they appear in the arc in air when the metal is vaporized, the evidence is strong that they belong to the oxide spectrum.

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A STUDY OF THE SPECTRUM OF α^2 CANUM VENATICORUM

By CAROL JANE ANGER

ABSTRACT

The spectrum of α^2 Canum Venaticorum has been studied by means of traces made on a Moll microphotometer, and the intensities of lines have been measured and plotted with a view to detecting periodic variations.

The existence of two groups of variable absorption lines has been confirmed, and periodic variation has been found for individual emission lines.

Periodic variation has been found for lines formerly considered steady and notably for lines not due to rare earths. The most important of these lines are Ca^+3933, Si^+4128, and Mg^+4481.

The wave-lengths of emission lines have been measured.

The spectrum of 12 α^2 Canum Venaticorum (1900.0 $\alpha = 12^h\ 51^m.4$, $\delta = +38°52'$) is of type Aop. The bases of the classification as peculiar were the strength of the silicon lines at λ 4128 and λ 4131 and the presence of numerous faint metallic lines "peculiar in wave-length."[1]

H. Ludendorff[2] discovered in 1906 that certain of the lines are variable in intensity. A. Belopolsky[3] in 1913 found two groups of lines, variable in intensity, and some in displacement, with a period of 5.50 days. C. C. Kiess[4] confirmed and extended his work. The lines of one group have been identified by F. E. Baxandall[5] as belonging to the element europium; Kiess provisionally identified the second group with terbium.

Recently, Belopolsky[6] has announced the presence of emission lines[7] (the total number of which was variable with the period of the absorption lines) and published a table of their wave-lengths. At

[1] Miss Maury, *Harvard Annals*, **28**, 96, 1897.

[2] *Astronomische Nachrichten*, **173**, 1, 1906.

[3] *Ibid.*, **196**, 1, 1913; also, *Bulletin de l'Académie Imp. des Sciences de St. Petersbourg* (6th ser.), **7**, 689, 1913.

[4] *Publications of the Detroit Observatory, University of Michigan*, **3**, 106, 1923.

[5] *Observatory*, **36**, 440, 1913; *Monthly Notices*, **74**, 32, 1913.

[6] *Astronomische Nachrichten*, **234**, 5, 1928.

[7] The presence of emission lines was noted by the writer at the Dearborn Observatory prior to the receipt of Belopolsky's announcement.

the same time he reported that the period was changing from $5^{d}.4705$ in 1913–1914 to $5^{d}.4695$ in 1919–1920 and further, to $5^{d}.4687$ in 1928.

P. Guthnick and R. Prager[1] found that the star was variable in light, with a range of 0.051 magnitudes in the period given by Belopolsky.

All of the observations of intensity up to this time have been visual estimates. The standard used by Kiess was λ 4128. It seemed that a study of the spectrum by mechanical methods would be particularly valuable. Accordingly, in the present investigation the registering microphotometer has been used as a means of studying the intensities objectively.

Sixty-seven plates, taken with the single-prism spectrograph attached to the $18\frac{1}{2}$-inch refractor of the Dearborn Observatory, were available. Fifty were made on Eastman 40, eight on Cramer Instantaneous Isochromatic, and nine on Eastman Speedway plates. The width of the spectrum is about 0.55 mm. The exposure time was about thirty-five minutes. Microphotometer tracings were made of forty-nine of these plates, which had been taken over a period of seventeen months. Through the courtesy of the Director of the Ryerson Physical Laboratory of the University of Chicago, the use of their Moll microphotometer was extended to the writer, who made fifty traces with it. Seven were made by Mr. C. T. Elvey at the Yerkes Observatory on the microphotometer of his design. In all, fifty-seven traces were made, the duplicates giving a check on the performance of the different instruments and of the same instrument on different days.

On each microphotogram a trace was made of a standard intensity strip, which had been exposed on the plate by means of a rotating sector. A relation could thus be obtained between a known change of intensity in stellar magnitudes and a given deflection of the galvanometer. The measures of intensity were derived by ruling the sensitometer steps across the face of the trace and reading the deflections in terms of hundredths of a division by means of a transparent divergent scale. Observations were made on all lines whose intensity was great enough to insure sufficient

[1] *Berlin-Babelsberg*, 1, 44, 1914.

accuracy in their measurement. Readings were taken from the continuous spectrum to the maximum deflection caused by the given line, no attempt being made to measure areas; for this reason the slit of the thermopile of the microphotometer had to be made very narrow. The position of the continuous spectrum was sometimes difficult to locate. Two completely independent sets of measures were made on each trace and the mean taken for the final value for the plate. The agreement between the two sets was very close.

The duplicate traces gave satisfactory results; those made on the two different instruments, and on the same instrument at different times, were accordant. Moreover, it is worth noting that almost identical traces were given for plates taken on the same night but presenting to the eye somewhat different qualities.

Heavily exposed plates could be used for dependable measures only in the violet; those that were too lightly exposed also tended to be unreliable. Aside from this, the traces were not all of the same quality and the cause of difference could not always be determined. Frequently it seemed to lie in the plate. The entire group of Speedway plates had to be given a low weight. They showed such coarse grain structure that it was almost decided to omit them without making any traces. They were included, however, in the hope that the machine's integration over the width of the spectrum would annul the irregularities. In the graphs which accompany this paper they are shown by crosses instead of dots. In other cases where there was an a priori reason for distrusting a value, it is plotted with a cross. In general, the observations scattered somewhat, but frequently extremely gratifying results were obtained. For example, plates of the same phase, taken over a period of months, often gave almost identical values.

The traces were first examined qualitatively for general and obvious differences. One of the most striking phenomena was the behavior of the silicon lines λ 4128 and λ 4131. On some of the plates λ 4128 was more intense than λ 4131; on some they were equal; and on others λ 4128 was less intense than λ 4131. The prominence of λ 4131 was connected with the prominence of λ 4205, one of the most conspicuous variable lines; similarly, λ 4128 was related to λ 4233. A line at λ 4133 was also noted; at maximum

intensity it was nearly as strong as the silicon lines, while many times it could not be seen at all. In the lists of Belopolsky and Kiess appear λλ 4205, 4233, and 4133.

On certain of the plates a group of absorption lines with beautiful regularity of intensity and position appeared between $H\beta$ and $H\gamma$. One of these was the magnesium line λ 4481. It was evident that the rare earths, especially europium and terbium, could not account for all the variations—a discovery contrary to prevailing opinion.[1]

A dip in the continuous spectrum of some of the traces indicated a variable absorption band in the λ 4200 region. This band, which was not present on all plates, extended from approximately λ 4178 to λ 4210.

Photographic reproductions of two traces accompany this paper (Fig. 1). The variations in λλ 4128, 4131, 4133, 4205, 4233, 4481, etc., are very clearly shown.

As the studies were extended, it became apparent that many lines, both emission and absorption, were variable. Their intensities were plotted, using the published values of the period. The period 5.50 days did not give satisfactory results; the variations of many lines could be represented with Belopolsky's period, 5.4687 days. The time, seventeen months, covered by these plates was too short to discriminate between his several periods, which vary in the thousandth of a day.

The presence of the two groups of absorption lines, as noted by Belopolsky and Kiess, was easily confirmed. The most prominent representative of group I was λ 4205; at maximum intensity it was as strong as the silicon lines; at minimum, it almost disappeared. The curve obtained is shown in Figure 2, plotting actual observations. Ordinates represent intensities in stellar magnitudes; abscissae, phase in days. One sensitometer step equals 0.38 magnitudes. The range of variation for this line is 0.205 magnitudes. The probable error of a single observation was obtained by measuring the deviation of each observation from the curve; it was found to be ±0.019 magnitudes for this line.

The second group is best represented by λ 4233. This line,

[1] Henroteau, *Handbuch der Astrophysik*, **6**, 362, Berlin, 1928.

however, was a blend with the enhanced iron line at λ 4233.3[1] and never entirely disappeared. Figure 3 is the curve for this line. The rather abrupt rise and fall of the curve should be noted—also,

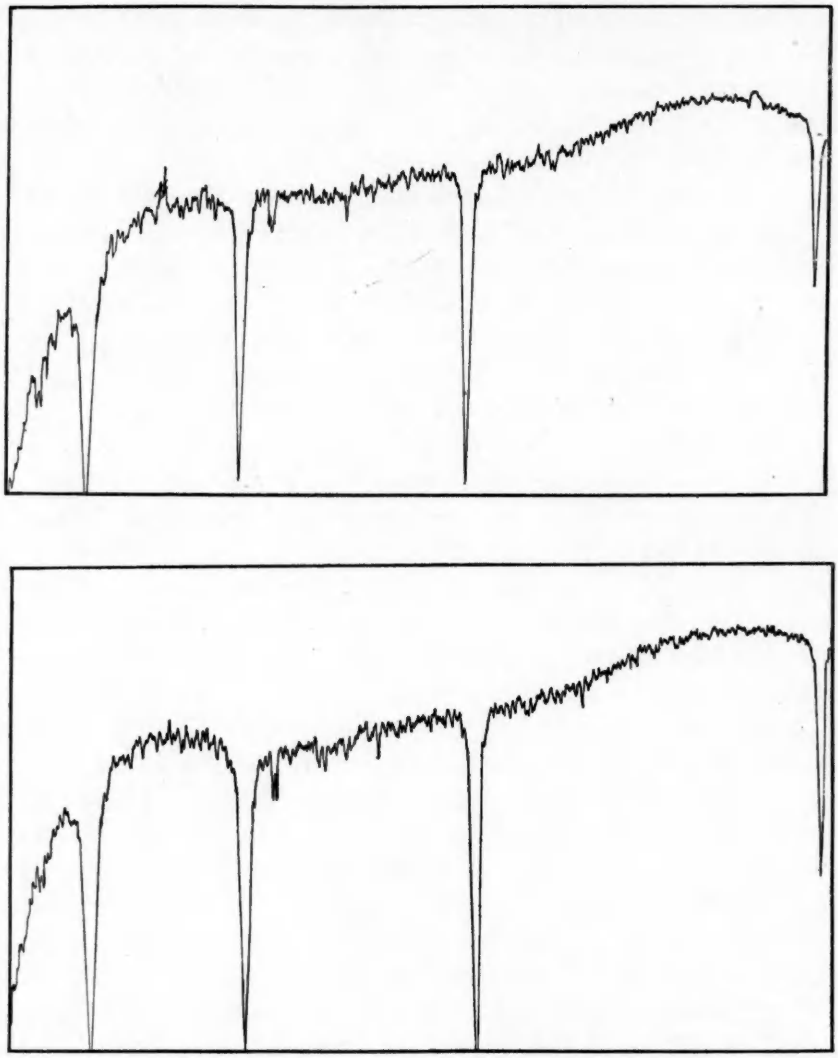
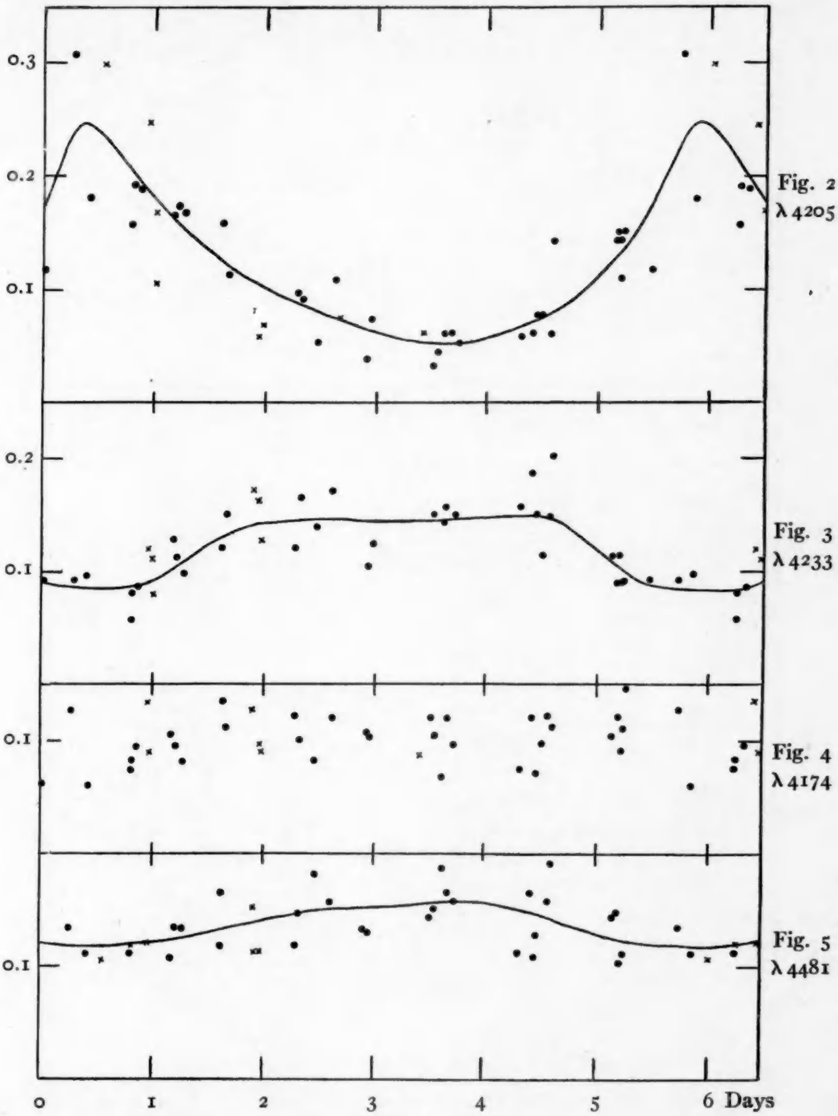


FIG. 1.—Density-curves of the spectrum of α^2 Canum Venaticorum

[1] Kiess, *loc. cit.*

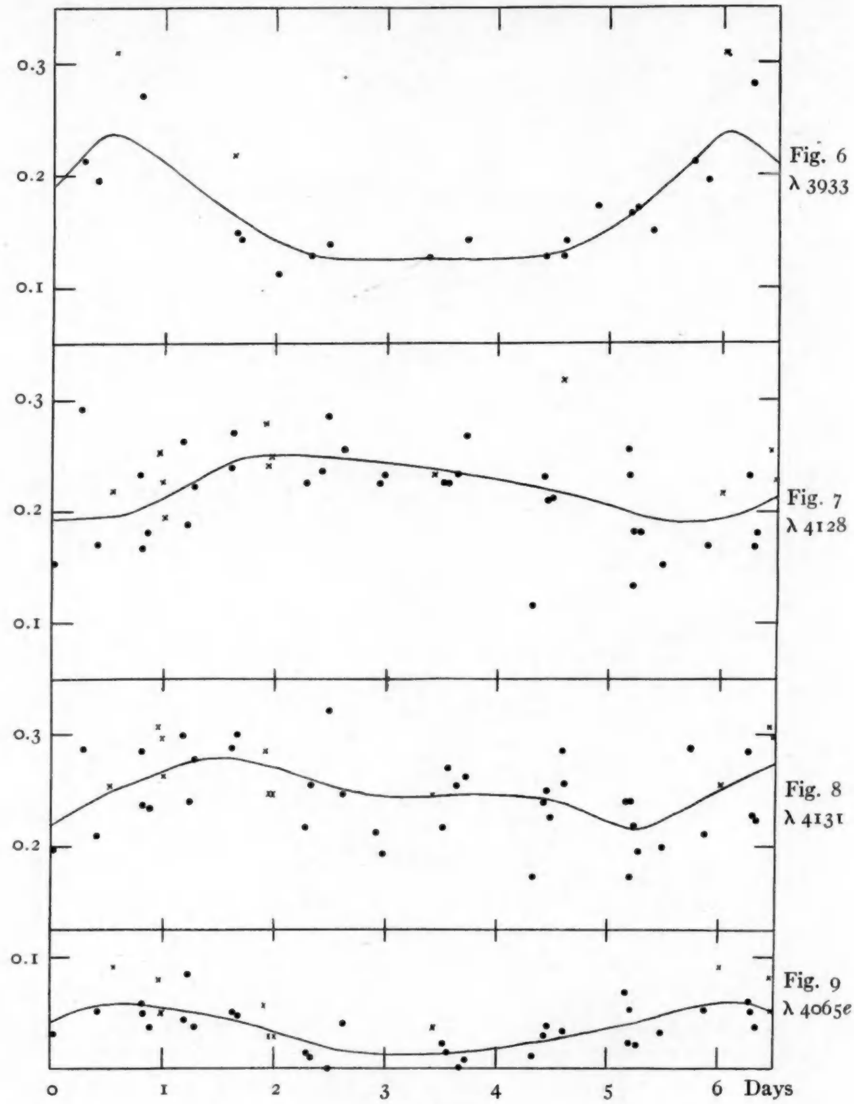
the fact that the curve of λ 4233, representative of group II, is the reflection of that of λ 4205, of group I.



FIGS. 2–5

Not all the lines investigated fell into these two groups. Some (Fig. 4) gave no evidence of periodicity. This was at least not en-

tirely due to inaccurate measures, for in many cases more prominent lines (i.e., giving greater galvanometer deflections) did not respond to the period which satisfied less prominent lines. A list



FIGS. 6–9

TABLE I

VARIATIONS OF ABSORPTION AND EMISSION LINES

					Mg.
3930.4	*Eu*	a	19	Group I	0.137
3933.7	*Ca*	a	19	Group I	.119
3945.9	......	e	14	Group I	.061
3988.9	......	e	19	Group I	.038
3991.4	......	a	25	Group I	.068
4000.4	......	a	26	No evidence of periodicity	.....
4002.6	*Tb*	a	30	Group II	.053
4012.4	*Tb, Eu*	a	34	No evidence of periodicity	.....
4019.2	......	e	27	No evidence of periodicity	.....
4023.2	......	e	20	No evidence of periodicity	.....
4024.6	*Dy, Ti*	a	35	No evidence of periodicity	.....
4032.8	......	a	32	Group II	.042
4046.4	......	a	39	No evidence of periodicity	.....
4048.8	......	a	40	Group I	.061
4050.3	*La, Dy*	a	42	Group I	.053
4055.2	......	a	39	Group II	.042
4063.4	*Gd*	a	36	No evidence of periodicity	....
4065.4	......	e	35	Group I	.053
4068.3	......	e	35	Group I	.053
4075.5	......	a	38	Group II	.042
4077.7	*Yt, Dy*	a	37	No evidence of periodicity	.....
4078.9	......	e	34	Group I	.023
4128.0	*Si*	a	38	Group II	.061
4130.9	*Si*	a	39	Group II?	.....
4132.5	......	a	32	Group I	.144
4134.7	......	e	34	Group I?	.....
4146.2	*Dy*	a	35	No evidence of periodicity	.....
4168.6	......	e	34	No evidence of periodicity	.....
4172.0	Ti^+, *Ga*	a	39	No evidence of periodicity	.....
4173.6	......	a	39	No evidence of periodicity	.....
4179.1	......	a	39	Group II	.038
4190.8	......	a	37	No evidence of periodicity	....
4200.7	*Tb*	a	39	Group II	.072
4205.1	*Eu*	a	41	Group I	.205
4221.6	......	e	33	Group I	.042
4233.1	*Tb*, Fe^+	a	38	Group II	.072
4261.9	......	a	35	Group I	.034
4269.3	......	a	33	Group I	.068
4382.4	......	e	32	No evidence of periodicity	.....
4384.1	......	a	37	Group II	0.042

TABLE I—*Continued*

					Mg.
4407.8	......	a	36	Group I	0.061
4435.3	......	a	35	Group I	.008
4450.4	......	a	37	Group I?	.....
4481.2	Mg^+	a	37	Group II	.042
4493.5	......	a	35	No evidence of periodicity	.....
4515.5	......	a	35	No evidence of periodicity	.....
4520.2	......	a	34	No evidence of periodicity	.....
4522.6	*Eu, Ti*	a	34	Group I	.038
4549.4	Fe^+	a	37	Group II	.049
4555.4	......	a	36	Group II	.049
4558.6	Cr^+, *La*	a	36	Group II	.057
4584.0	*Fe*	a	35	No evidence of periodicity	.....
4621.6	......	a	35	Group II	0.042

Wave-lengths and identifications of absorption lines are taken from Kiess, reduced to I.A. The fourth column gives the number of times observed; the sixth, range of variation in stellar magnitudes.

of lines investigated and their classification will be found in Table I.

Some of the lines between $H\beta$ and $H\gamma$, which had attracted attention, were found to belong in each of the groups, and some showed practically no evidence of periodicity. Of first importance is the detection of periodic variation, typical of group II, in the magnesium line λ 4481 (Fig. 5). This line had been reported by Ludendorff to change its character, but was measured by Belopolsky and Kiess as a constant line. Equally important is the discovery of periodic variation of the calcium line λ 3933, belonging to group I (Fig. 6).

It was a matter of especial interest to study the peculiarities of the silicon lines. As has been remarked, their unusual strength has furnished a part of the basis of the classification of the spectrum as peculiar. To the second group of variable lines λ 4128 (Fig. 7) seems to belong. The changes in λ 4131 (Fig. 8) may be attributed in part to a variable line, λ 4130, not resolved on our plates. This line, whose variability was discovered by Belopolsky, belongs to group I. Consequently, variations due to this line should yield a curve similar to that of λ 4205; such a curve was not found for λ 4131. On the other hand, the measures are such as might be given

TABLE II

Wave-Lengths of Emission Lines

3945.9	v	2	4206.6	v	2	4397.5	r	2
3949.5	..	2	4212.4	r	2	4403.9	..	2
3955.5	r	2	4214.3	v	2	4412.3	bv	3
3983.0	vr	2	4216.6	vr	2	4415.7	vr	1
3985.2	vr	2	4221.6	b	2	4429.2	r	2
3988.9	r	4	4226.5	vr	2	4433.0	bv	4
4001.4	vr	2	4228.5	...	2	4437.3	r	2
4010.0	v	2	4231.5	...	4	4442.4	v	3
4019.2	vr	2	4234.9	v	2	4446.6	r	4
4023.2	vr	4	4237.2	r	2	4464.1	..	1
4027.2	b	2	4239.9	v	3	4470.2	r	1
4031.6	vr	2	4249.3	r	4	4472.4	v	1
4035.1	r	2	4251.9	v	2	4478.3	..	1
4047.6	vr	2	4257.3	v	3	4479.6	r	2
4059.6	r	3	4263.6	r	3	4483.0	bv	3
4065.4	..	2	4268.4	v	2	4495.5	..	2
4068.3	..	1	4274.6	r	3	4498.3	r	4
4069.4	r	2	4276.9	v	3	4512.0	..	2
4072.1	..	1	4279.2	r	1	4513.5	r	1
4074.5	r	4	4287.3	...	2	4518.2	vr	4
4078.9	v	4	4291.8	bv	1	4521.7	vr	2
4125.9	br	3	4295.4	r	2	4524.8	bv	2
4134.7	..	2	4298.3	v	4	4532.1	r	2
4140.2	v	1	4301.9	vr	3	4548.4	r	2
4158.0	v	1	4305.0	...	3	4574.4	r	2
4159.5	br	3	4311.1	...	3	4578.1	v	2
4163.1	r	3	4319.1	b	1	4581.2	..	1
4168.6	vr	3	4323.2	bv	2	4585.5	v	3
4170.1	r	2	4328.6	bvr	2	4614.8	r	2
4176.7	br	3	4350.6	r	3	4623.8	v	4
4180.9	v	2	4353.6	vr	2	4631.8	v	3
4185.8	vr	4	4359.1	...	1	4656.4	vr	4
4192.4	v	1	4382.4	r	3	4682.6	r	2
4193.2	r	2	4389.0	vr	4	4734.2	..	1
4199.6	vr	2	4392.7	vr	3	4760.5	v	2
4204.1	r	3	4394.2	r	1	4782.3	..	1

The first column gives the wave-length in international Angstrom units. In the second column, "b" indicates broad; "v" absorption to violet; "r" absorption to red. The third column gives the number of times measured.

by a line which varied as the lines of group II and which was more or less confused with a line of group I.

The emission lines, causing smaller galvanometer deflections and of opposite sign, were more difficult to measure accurately. In spite of this fact, some gave a very definite periodic curve (Fig. 9). All the periodic emission lines found belong to group I.

As many emission lines as could be observed were measured for wave-length on four plates. Instead of measuring two classes of emission—bright lines and lines with bright edges—as Belopolsky did, in this study the actual emission was measured in both cases. It was frequently found on either or both sides of an absorption line, and in some cases isolated. The reduction to the sun was made on all four plates, but no correction was applied for the velocity of the star, inasmuch as the various determinations yield values very close to zero, and both positive and negative. They are summarized by Kiess as follows:

Pulkovo	−6.5 km/sec.
Potsdam	−0.3
Lick, Campbell	−1.6
Lick, Kiess	+0.3
Ann Arbor	−0.87±0.37 km/sec.

No completely satisfactory explanation of the peculiarities has been advanced. Belopolsky proposed a gaseous ring or satellite in orbital motion about the central nucleus. Kiess pointed out that this hypothesis, if the identifications of the lines are correct, would require separate condensations of europium and terbium. Though it now appears that lines of other elements are variable, the difficulty is still present; this explanation would require a separation of the elements.

A simpler explanation was proposed by A. Fowler,[1] who suggested that the levels of the gases might change sufficiently to cause the variation.

Alpha Canum Venaticorum, moreover, shows some of the characteristics of the Cepheid variables, and perhaps the solution of its peculiarities might well be connected with the problem of the

[1] *Observatory*, **36**, 461, 1913.

Cepheids. The principal difficulty would be encountered in explaining the presence of more than one group of variable lines.

In conclusion, I wish to express my thanks to Professors Philip Fox and Oliver J. Lee for valuable suggestions and criticisms; to Professor George S. Monk, of the Ryerson Physical Laboratory of the University of Chicago, for his kindness in making the Moll microphotometer available to me; and to Mr. C. T. Elvey for the traces which he made at the Yerkes Observatory.

DEARBORN OBSERVATORY
EVANSTON, ILLINOIS
July 2, 1929

NOTE

While this paper was in the office of the editor, it had the advantage of an examination by Dr. O. Struve of the Yerkes Observatory. He has kindly called to the author's attention an important investigation on this star which was unavailable to the author because it was printed in the Russian language. Résumé of this work and his comment upon it follows.

An important series of papers on the variation of the absorption lines in α^2 Canum Venaticorum appeared in *Bulletin de l'Observatoire Central à Poulkovo*, **11**, 2 (No. 101), 1927. B. P. Gerasimovič discusses the variation of certain lines from spectrograms taken by A. Belopolsky in 1915–1916. V. A. Rossovskaya included the same observational material and extended the work to cover the plates obtained by Belopolsky in 1917. Finally, A. V. Markov has rediscussed the whole material collected at Poulkovo between 1910 and 1920. The intensities were measured by means of a scale, and the results thus obtained were compared with ordinary visual estimates. Variation of the type of group II of the present paper was recorded for the lines $\lambda\lambda$ 4003, 4123, 4128, 4131, 4174 (?), 4201, 4234, 4261 (?), 4297 (?), 4304 (?), 4326, 4353 (?), 4404 (?), 4416, and 4481. Variation of the type of group I is shown by the following lines: $\lambda\lambda$ 4130, 4132, 4168 (?), 4296, 4483 (?). Constant intensity was found for the following lines: $\lambda\lambda$ 4013, 4033, 4049, 4125, 4165, 4171, 4172, 4178, 4414, 4415. It is noted that the variation of the

lines *Si* 4128, *Si* 4131, and *Mg* 4481 is recorded for the first time. The period usually assumed is $5^{d}.4705$. From a discussion of the maxima of intensities for the whole range of observations it is found that this value needs a correction of $-0^{d}.0010 \pm 0^{d}.0005$. The mean radial velocity for 1919–1920 was -4.57 km/sec. Variations in the width of certain lines and several cases of doubling are also noted.

All the evidence on the variation of spectral lines in various stars, collected at Poulkovo and elsewhere, are discussed by Belopolsky. It is found that the variations of line intensities in α^2 Canum Venaticorum are of the same type as in the Cepheid variables. Certain lines reach their maxima at the time of maximum light, other lines have their minima at the same phase, and finally there are lines which show several maxima and minima during one period of light, or of velocity.

It will be seen that the results of the Russian investigations are in good agreement with those of the foregoing study. The lines attributed by Markov to the two groups agree very well with the microphotometric results. In a few cases there is a discordance as to whether a line is variable or not, but since in these cases we are always concerned with small changes in intensity, exceptional cases of disagreement should not be surprising.

The Poulkovo plates were taken with a three-prism spectrograph and do not cover the part of the spectrum in the region of λ 3900. Consequently, the line *Ca K*, λ 3933, for which a variation like that of the I group is found here, has not been included in that work.

THE RADIAL VELOCITY OF β CEPHEI ON AUGUST 21, 1928

By JOHN C. DUNCAN and HELEN M. MITCHELL

ABSTRACT

From thirty-three spectrograms of β Cephei, obtained with the 60-inch reflector at Mount Wilson by Duncan on August 21, 1928, and measured at the Whitin Observatory by Miss Mitchell, the velocity-curve appears to be a *sine curve* of *semi-amplitude 15.5 km/sec.*, with a superposed *secondary oscillation* of semi-amplitude *2.2* km/sec., and *period half that of the main curve*. The *velocity of the center of mass*, earlier shown by Crump to be variable, is *−0.66 km/sec.*, the highest so far published. The *period*, which up to 1921 had appeared to be constant at 0.1904795 day, has certainly *changed*, and the observations here discussed may be brought into harmony with the Lick observations of 1921 by a period of either 0.1904838 or 0.1904722 day.

The variability of radial velocity of β Cephei (α 21^h $27^m.4$, $\delta+70°$ $7'$, 1900.0; magnitude 3.3; type B1) was discovered by W. S. Adams[1] at the Yerkes Observatory in 1901. In 1906 E. B. Frost[2] found the period to be four and one-half hours, one of the shortest ever determined ($0^d.1904$). From observations in 1912 C. C. Crump[3] found changes in the amplitude and in the velocity of the center of mass, and noted a secondary oscillation. In 1920 F. Henroteau[4] confirmed the variability of the velocity of the center of mass and appeared to establish the period as constant at 0.1904795. P. Guthnick[5] and Miss E. Cummings[6] have studied the star with photoelectric photometers and have found a minute variation of light in the period of the velocity variation and have shown that the interval between light-maximum and velocity-minimum is brief, making it probable that the star may be classed as a Cepheid variable.

On the night of August 21, 1928, thirty-three spectrograms of β Cephei were secured by Duncan with the 60-inch telescope and one-prism spectrograph at the Mount Wilson Observatory. A camera of 40-inch focal length was used which gave a dispersion of about 20 A

[1] *Astrophysical Journal*, **15**, 340, 1902.

[2] *Ibid.*, **24**, 259, 1906.

[3] *Detroit Observatory Publications*, **2**, 144, 1916.

[4] *Publications of the Dominion Observatory at Ottawa*, **5**, 77, 1921.

[5] *Handbuch der Astrophysik*, **6**, 222, 1928.

[6] *Lick Observatory Bulletins*, **11**, 120, 1923.

per millimeter. These spectrograms were measured and reduced by Miss Mitchell at the Whitin Observatory of Wellesley College. The wave-lengths of the star lines, which were kindly furnished us by Dr. A. H. Joy of the Mount Wilson Observatory, are given in Table I. Those of the iron comparison lines were taken from the revision of Rowland's tables. Table II gives the times of observation and the results of the measures. The time is Pacific standard time, counted from noon of August 28.

A velocity-curve was drawn, and preliminary elements were derived by the method of Lehmann-Filhés. As the eccentricity was

TABLE I

λ	Element	λ	Element
4267.140	C^{++}	4414.888	O^{++}
4317.160	O^{++}	4416.974	O^{++}
4319.647	O^{++}	4437.549	He
4340.467	$H\gamma$	4471.477	He
4345.57	O^{++}	4552.654	Si^{+++}
4347.429	O^{++}	4567.872	Si^{+++}
4349.435	O^{++}	4650.75	C^{+++}
4351.275	O^{++}	4713.143	He
4366.906	O^{++}	4861.326	$H\beta$
4387.928	He		

very small, in harmony with the results of earlier observers, it was assumed to be 0.00; the observations were grouped into eleven normal places, and a least-squares solution was made for the elements K, γ, and T, the last being the time of maximum positive velocity instead of the time of periastron passage. The resulting elements are given in Table III, and the residuals from the individual observations in the last column of Table II. The probable error of a single observation was found to be ± 1.81 km/sec.

The residuals from the normal places showed a consistent alternation from positive to negative, indicating a secondary oscillation. The semi-amplitude of this oscillation is 2.2 km/sec., and its period is half that of the main curve. It is recalled that Crump also found a secondary oscillation, but with semi-amplitude 1.25 km/sec. and a period one-third that of the star instead of one-half.

Crump's inference of variability of K and γ is confirmed; the

various values of these elements so far published are shown in Table IV. We are not in agreement, however, with earlier observers

TABLE II

Plate No.	P.S.T. Aug. 21, 1928	Exposure	No. of Lines Measured	V	O.−C.
				km/sec.	km/sec.
γ 16073	$7^{h}800$	10^{m}	14	+ 1.4	+0.5
16075	8.150	8	14	+ 7.2	−0.7
16076	8.383	14	12	+ 9.3	−2.2
16077	8.633	12	16	+11.5	−2.5
16078	8.883	14	16	+15.6	+0.8
16079	9.167	14	15	+13.9	+0.4
16080	9.450	14	15	+14.6	+4.5
16081	9.733	16	15	+ 6.2	+1.2
16082	9.967	6	14	+ 5.8	+5.6
16083	10.100	6	16	− 5.3	−2.7
16084	10.300	14	15	− 5.0	+1.8
16085	10.567	14	15	−10.7	+0.8
16086	10.850	14	16	−16.4	−1.5
16087	11.050	6	13	−19.9	−3.9
16088	11.250	12	15	−15.5	+0.5
16089	11.450	6	15	−12.5	+2.3
16092	12.167	10	16	+ 0.6	+3.8
16093	12.383	6	16	− 1.9	+0.8
16094	12.550	6	15	+ 3.3	−1.6
16095	12.750	6	16	+10.4	+1.8
16097	13.433	8	14	+16.0	+1.2
16099	13.783	10	16	+18.1	+5.1
16100	13.967	6	15	+10.1	−0.6
16101	14.183	14	16	+ 6.6	−0.5
16102	14.400	6	16	+ 6.6	+3.8
16103	14.533	6	17	− 1.8	−1.8
16104	14.667	6	15	− 6.4	−3.6
16105	14.867	8	16	− 8.0	−1.1
16106	15.033	6	17	−14.6	−4.6
16107	15.233	14	17	−11.3	+1.7
16108	15.467	6	14	−16.6	−1.3
16109	15.667	10	17	−18.3	−2.1
16111	16.283	14	14	− 8.5	+3.0

TABLE III

Semi-amplitude of velocity-curve .... $K = 15.51 \pm 0.31$ km/sec.
Velocity of center of mass .... $\gamma = -0.66 \pm 0.43$ km/sec.
Time of maximum positive velocity ...... $T =$ Aug. $21^{d}\ 8^{h}\ 52^{m}0 \pm 0^{m}08$ P.S.T.
= Aug. 22 4 52.0 G.C.T.
Eccentricity .... $e = 0.00$
Projected semi-major axis .... $a \sin i = 40{,}624$ km
Mass function .... $\dfrac{m^3 \sin^3 i}{(m+m_1)^2} = 0.00007\ \odot$
Period .... $P = 0.19047+$ day

in regard to the constancy of the period. Henroteau, from material published up to 1920, concluded, as had Crump, that the period was constant at 0.1904795 day, and this value satisfied the observations made in 1921 at the Lick Observatory;[1] but this period implies 16,390.4 cycles between the observed maxima of 1921 and 1928,

TABLE IV

Date	K	γ	Authority
1906 July 6	16.9	− 7.3	Frost*
1912 April 29	....	12.	Frost*
1912 Aug. 5	15.8	14.13	Crump
1912 Oct. 1	19.22	12.88	Crump
1914 Oct	16.8	26.1	Gerasimovič†
1916 Sept.–Oct.	19.0	32.3	Belopolsky‡
1917 Sept.–Oct.	18.0	22.5	Belopolsky‡
1919 Sept	11.3	9.8	Van Arnam§
1921 Nov. 6	16.0	2.0	Lick Observatory‖
1928 Aug. 21	15.51	− 0.66	Present paper

* *Astrophysical Journal*, **64**, 27, 1926.
† *Bulletin de l'Observatoire Central de Russie*, **7**, 80, 1917.
‡ *Bulletin de l'Académie des Sciences de Russie*, No. 16, p. 1803, 1918.
§ *Popular Astronomy*, **36**, 348, 1928.
‖ *Lick Observatory Publications*, **16**, 315, 1928.

which is impossible since the number of cycles between maxima must be an integer. The discrepancy is too large to be attributed to errors of observation. Assuming the actual number of cycles between the observed maxima of 1921 and 1928 to be 16,390, the average period for that interval must be 0.19048381 day; assuming 16,391 cycles, the average period comes out 0.1904722 day.

Whitin Observatory
Wellesley, Mass.
June 1929

[1] *Lick Observatory Publications*, **16**, 315, 1928.

THE STELLAR CALCIUM LINES IN SPECTRAL TYPES A AND B

By O. STRUVE and C. D. HIGGS

ABSTRACT

The *contour of the calcium line K* was determined in four stars of class A. *For α Lyrae* the observed contour *agrees* well *with the theoretical formula of Unsöld*. The total number of atoms $N \cdot D = 3.5 \times 10^{16}$. For α Cygni the agreement is not so good. An approximate fit gives $N \cdot D = 10^{17}$. The lines in α Aquilae and α Ophiuchi are wide and shallow and cannot be represented by the Unsöld formula. The *total number of atoms $N \cdot D$ was determined* also *from estimates of the central intensity of the K line* in a large number of stars. The values range from 4×10^{16} for type A2 to 3×10^{15} for type B5.

In stars of class A and B the stellar Ca^+ lines H and K are so narrow that their contours cannot be determined with spectrographs of small dispersion or with objective prisms. In Vega the width of K on our one-prism spectrograms is about 0.5 A, which agrees with the limit imposed by the finite resolving power of the instrument. We have recently obtained a number of plates with the Bruce spectrograph arranged for two prisms, giving a linear scale of 6.46 A/mm at K. The contours of Ca^+ K for four stars are shown in the table.

The contour of α Lyrae agrees well with Unsöld's theoretical curve. For α Cygni the observed contour is somewhat wider near the middle of the line and falls off more rapidly near the edges. We have adjusted the variable $N \cdot D$ in Unsöld's formula, designating the number of active atoms in the reversing layer, to represent the contours of these two stars:

$$\alpha \text{ Lyrae}: N \cdot D = 3.5 \times 10^{16} ,$$

$$\alpha \text{ Cygni}: N \cdot D = 1.0 \times 10^{17} .$$

The contours of α Aquilae and α Ophiuchi cannot be represented by this formula. The lines are too wide and shallow and resemble the "dish-shaped" helium lines observed by Elvey and attributed by him to axial rotation.

For the determination of $N \cdot D$ in stars of early spectral type which could not be photographed with high dispersion, we used total

CONTOUR OF Ca^{+} *K*

PERCENTAGE ABSORPTION

$\Delta\lambda$	α Lyrae	α Cygni	α Aquilae	α Ophiuchi (Uncertain)
0.00	74	80	75	49
.09	71	80		
.19	64	78	73	49
.28	48	76		
.37	34	72	72	48
.46	24	62		
.56	16	49	70	47
.65	11	40		
.74	08	29	68	46
.84	07	24		
0.93	05	16	65	45
1.02	01	13		
.11	00	11	61	42
.21	00	10		
.30	00	09	58	40
.39	00	08		
.48	00	06	54	38
.58	00	05		
.67	00	04	50	34
.76	00	03		
.86	00	02	49	32
1.95	00	01		
2.04	00	01	46	27
.13	00	00		
.23	00	00	44	22
.32	00	00		
.41	00	00	41	18
.51	00	00		
.60	00	00	37	16
.78	00	00	34	14
2.97	00	00	31	12
3.16	00	00	30	08
.34	00	00	28	07
.53	00	00	26	06
.71	00	00	22	04
3.90	00	00	20	04
4.08	00	00	18	02
.27	00	00	18	02
.45	00	00	16	01
.64	00	00	13	01
4.83	00	00	12	00
5.01	00	00	11	00
.20	00	00	10	00
.38	00	00	08	00
.57	00	00	08	00
.75	00	00	07	00
5.94	00	00	06	00
6.12	00	00	04	00
.31	00	00	02	00
.50	00	00	02	00
.68	00	00	01	00
6.87	00	00	00	00

absorbed energies, rather than the actual contours. Assuming that the true contour remains of the Schwarzschild-Schuster approximation and that the absorption coefficient is that of Unsöld, we have, in the usual notation:[1]

$$E_{th}=\int_{-\infty}^{+\infty}\left\{1-\frac{1}{1+\frac{C\cdot N\cdot D}{(\lambda-\lambda_0)^2}}\right\}d\lambda=\pi\sqrt{C\cdot N\cdot D}\,.$$

The observed contour is that produced by the optical qualities of the instrument, which in this case was the one-prism Bruce spectrograph. The contour is approximately an error-curve, the absorbed energy being redistributed in such a way that the width of the line remains approximately constant. The contour of α Lyrae shows that the width, 0.5 A, is measured at the point where the intensity has fallen to one-half its maximum value. Assuming that this holds also for other stars, we find that the observed total absorbed energy is approximately given by[2]

$$E_{\text{obs}}=\int_{-\infty}^{+\infty}(1-I_c)e^{-\kappa^2(\lambda-\lambda_0)^2}d\lambda=\frac{(1-I_c)\sqrt{\pi}}{1.66}\,.$$

$N\cdot D$ is computed by making the theoretical and the observed expressions agree with one another:

$$N\cdot D=\frac{10^{17}(1-I_c)^2}{1.4(1-m)}\,.$$

The quantity m is a small correction introduced to take account of the deviation of Unsöld's curve in the middle of the line from the true contour. This is a slowly varying function of $N\cdot D$. For $N\cdot D=10^{19}$ it is about equal to 0.1, gradually increasing to about 0.2 for $N\cdot D=10^{15}$. In practice we compute $N\cdot D$ by two successive approximations. The correction $(1\text{-}m)$ is neglected at first. With the resulting value of $N\cdot D$ we compute $(1\text{-}m)$ and solve again for $N\cdot D$. The observational material consisted of estimates of the intensity of the K line in many stars. The means for each spectral subdivision

[1] *Monthly Notices of the Royal Astronomical Society*, **89**, 586, 1929.

[2] I_c is the observed central intensity.

are given below. The values in the second column are expressed in arbitrary units. These have been calibrated several times by different methods, and the respective values in percentage absorptions $(1-I_c)$ are given in the third column. The fourth column gives the final values of $N \cdot D$. The star α Lyrae has been added in order to compare the result with that obtained from the actual contour. The agreement is good. For the other types the values of $N \cdot D$, while subject to considerable uncertainty, probably indicate the correct order of magnitude.

Sp. Type	Intensity of K; Arbitr. Units	Percentage Absorption $(1-I_c)$	$N \cdot D$
A2	20.0	75	4×10^{16}
A0	8.5	51	1×10^{16}
B9	4.5	33	7×10^{15}
B8	3.8	28	5×10^{15}
B5	3.0	23	3×10^{15}
α Lyrae	10.0	55	2×10^{16}

YERKES OBSERVATORY
August 1929

MINOR CONTRIBUTIONS AND NOTES

VARIABLE HYDROGEN LINES IN 15 κ CASSIOPEIAE

In a recent survey of the spectra of B-type stars photographed with the Bruce spectrograph of the Yerkes Observatory it was

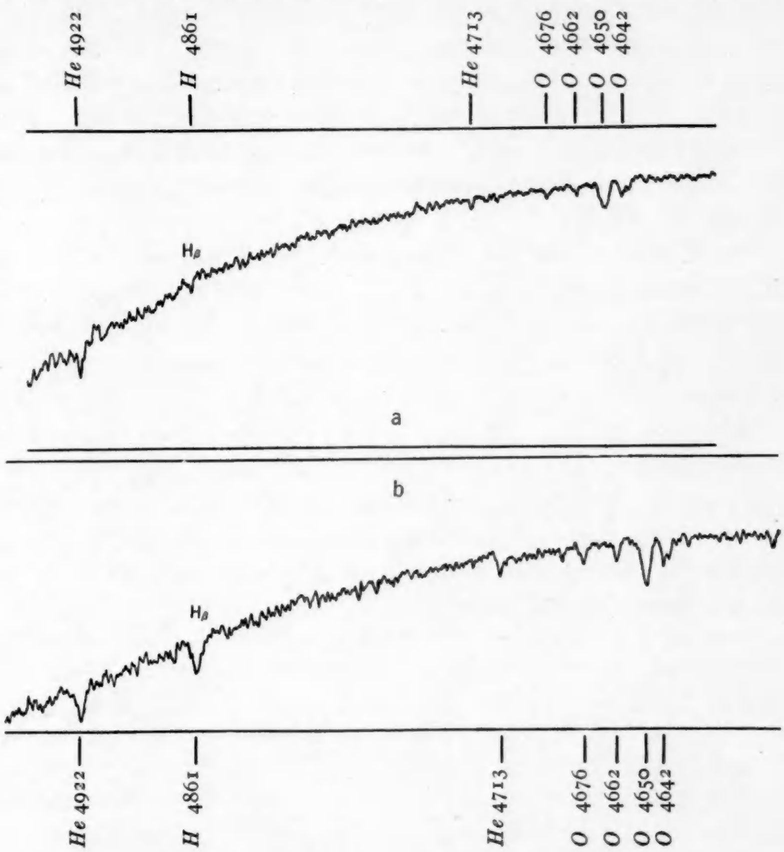


FIG. 1—Microphotometer tracings of the spectrum of 15 κ Cassiopeiae:
a) 1908 September 8, $15^h\ 10^m$ G.M.T. ($H\beta$: very faint)
b) 1908 October 5, $15^h\ 07^m$ G.M.T. ($H\beta$: strong absorption)
The parallel lines indicate complete darkness (top) and clear film (bottom)

noticed that the star 15 κ Cassiopeiae (α $0^h 27^m\!.3$; $\delta + 62^\circ 23'$; 1900) has variable hydrogen lines. The details of our six spectrograms

of this star are shown in the table. The accompanying figure contains microphotometer tracings of two of the plates. It will be noticed that the upper tracing, for September 8, 1908, shows hardly any depression at $H\beta$. There is probably a very slight emission on both sides of the normal position of the line, with a faint absorption line in the middle. The lower tracing, for October 5 of the same year, shows a well-developed absorption line at $H\beta$ without any apparent trace of emission. The helium absorption line λ 4922 is present in both tracings, and so are the oxygen lines $\lambda\lambda$ 4642, 4650, 4662, and 4676 as well as the helium line λ 4713. The line $He+$ λ 4686 can barely be discerned in the lower tracing. The emission to the red of *He* λ 4713 in the lower tracing is probably not real. It is of interest to note that the change in the appearance in $H\beta$ must have taken place in the short interval between September 8 and October 5, 1908.

The *Henry Draper Catalogue* gives the spectrum of this star as B0. Its photographic magnitude is 4.00 and its visual magnitude is 4.24. P. W. Merrill found it to be a spectroscopic binary, at the Lick Observatory.[1] No particulars concerning the appearance of the hydrogen lines are given. The orbit of this star has not been determined and the total range in velocity is probably not large.[2] I suspect that the width and the intensity of the narrow lines varies from plate to plate, probably in relation to the orbital period. However, the large variation in the appearance of $H\beta$ cannot be due to blending of two spectral components and must be attributed to physical changes in the star.

Date	G.M.T.	Description of $H\beta$
1908 Sept.	7.684	No depression in the continuous spectrum at $H\beta$
1908 Sept.	8.632	Probably slight emission with very weak absorption in the middle
1908 Oct.	5.630	Strong, normal absorption line
1910 Aug.	8.776	Weak and rather sharply bounded absorption line
1910 Aug.	12.824	Same as preceding plate
1929 July	2.313	Weak absorption line

YERKES OBSERVATORY
June 29, 1929

OTTO STRUVE

[1] *Publications of the Lick Observatory*, **16**, 6, 1928; *Lick Observatory Bulletins*, **6**, 141, 1911.

[2] *Astrophysical Journal*, **64**, 18, 1926 (star No. 6).

REVIEWS

Zeemaneffekt und Multiplettstruktur der Spektrallinien. By E. Back and A. Landé. Berlin: Julius Springer, 1925. Pp. xii+213. Plates 2; figs. 25. M. 14.40; bound, M. 15.90.

The series of monographs "Struktur der Materie," under the editorship of Max Born and J. Franck, was instituted some four years ago with the purpose of presenting briefly the most recent developments in the various special fields of this subject. As the first volume of this collection, of which seven have now been published, there appeared the foregoing monograph on the Zeeman effect. On reading it over in the light of subsequent additions to our knowledge of atomic spectra, one is struck by the numerous difficulties that have been cleared up, particularly by the advent of the new quantum mechanics. This does not materially detract from the value of the book, however, because of course the laws governing the spectral terms and their splitting in a magnetic field have not changed, although the theoretical interpretation of these laws has been fundamentally revised. Furthermore, the description of the experimental technique for the production and analysis of Zeeman patterns which has been developed by Professors Back and Landé in the laboratory at Tübingen contributes greatly to the permanent worth of this volume. It is the only authoritative and detailed work devoted exclusively to the Zeeman effect available at present.

In order to understand the complex phenomena of the anomalous Zeeman effect and of the Paschen-Back transition effect, one must first be familiar with the interpretation of multiplets in terms of the simple mechanical model of the atom. Hence the book begins with the theoretical treatment (by Landé,) giving first the classical and quantum theories of the normal Zeeman effect. Then follows a brief description of the empirical results for the anomalous effect, and of the resolution of the line-splittings into term-splittings. This leads naturally into a development of the systematics of term multiplicities. In this connection Landé's interval rule and the rules for the intensities of multiplet lines and their Zeeman components are developed with the aid of the correspondence principle and the familiar atomic model in which the angular momenta of the orbital electron, the whole atom, and the "core" are measured by quantum numbers K, J, and R. Unfortunately, the concept of the spinning electron

had not been introduced at the time of writing. Also, one regrets that the now-familiar term notation of Russell and Saunders, already proposed at that time, was not used instead of the awkward $n^r{}_{kj}$ scheme. A better treatment of certain aspects of the theory will be found in the fourth volume of this series (Hund, *Linienspektren*), where the results of the new quantum mechanics are incorporated, with the correct normalization of quantum numbers. This applies also to the sections on spectra in which Landé's g-formula no longer holds, such as that of neon, and on the Paschen-Back effect. Brief discussions of the relation between anomalous Zeeman effect and other phenomena of atomic magnetism, and of the origin of optical doublets ("relativistic" versus "magnetic") are given at the end of the theoretical division.

The part of the book that is undoubtedly most consulted at present is the second half, in which an excellent exposition of the experimental methods for studying the Zeeman effect is given. Of particular value is the detailed consideration of various sources of light and their properties in a magnetic field, including the vacuum-arc box, with which many beautiful results have been obtained. Following this, the production and measurement of the magnetic field are considered. Noteworthy also is the description of Paschen's original mounting for the 21-foot grating at Tübingen, an arrangement which is in several respects better than Rowland's, at least for mounting large concave gratings. At the end of the book there is a section on the practical analysis of Zeeman types, and it is shown how the difficulties met with—due, for example, to an incipient Paschen-Back effect—may be overcome. Appendices contain (1) literature in the interval 1914–1924, (2) g-factors in order of increasing numerical value, (3) theoretical Zeeman patterns from doublets to septets, and (4) description of two plates reproduced photographically from original plates of the Zeeman effect taken at Tübingen. These constitute a particular adornment to the volume, and are indicative of the splendid work these authors have done in this field.

F. A. JENKINS

Lichtelektrische Erscheinungen. By BERNHARD GUDDEN. "Structur der Materie in Einzeldarstellung" (ed. M. Born and J. Franck), Vol. III. Berlin: Julius Springer, 1928. Pp. ix+327. Figs. 127. Bound, M. 25.20.

This book gives an excellent account of photo-electric researches from 1914 to 1927, with some references to papers published in 1928. The descriptions of past work are clear and adequate, necessitating a mini-

mum of reference to the original papers. Supplementary comments and criticisms by the author give added interest and value to the work.

As explained in the Preface, emphasis is laid on the observed facts, and only such experimental technique and methodology are included as are necessary for a clear exposition of the results and of their importance. The theoretical aspects of the various phenomena are touched upon briefly for emphasis or explanation, but are not treated separately or at any length. One basis for the author's decision has been dissatisfaction with the old theoretical treatments and a modest disclaimer of expert knowledge of the new theoretical methods. The involved nature of the experiments in many cases precludes any simple theoretical analysis. For the cases of photo-ionization of gases, and photo-electric phenomena in the X-ray region, the basic laws are already familiar. Limitation of the book to the experimental exposition of the photo-electric effect has allowed the author to write in his own field and produce a highly readable text valuable to experimental and theoretical physicists as a source and reference book.

The subject is divided into four parts: (1) external and (2) internal photo-electric effect in solids and liquids; (3) photo-electric effect in gases; and (4) at high frequencies (X-ray and γ-ray regions). Emphasis is laid on the first two divisions, of which the first occupies 130 pages and the second 90 of the 280 pages of the whole book. There is one short chapter on photo-electric cells and their scientific uses. One chapter is given to the photo-electric effect in gases; and another of the sixteen chapters is devoted to phenomena in the X-ray and γ-ray region, including the Compton effect. Sections on phosphorescence, fluorescence, photochemistry, critical potentials, Becquerel effect, phototropy, and the Weiger effect are included, to round off the treatment.

There is a bibliography of 633 papers published in the period from 1914 to 1928. These papers are arranged alphabetically for each year and are designated by consecutive numbers which also appear in the text for each reference. In the Bibliography, page numbers are given for every reference in the text. In addition, there are separate indices for authors, subjects, and specifically mentioned photo-electric substances. There are also included tables of physical constants, of equivalent energies, and a periodic table. Subheadings of each chapter with their page references are given in the Table of Contents and furnish easy access to any desired section.

K. T. Bainbridge

The World of Atoms. By ARTHUR HAAS. Translated by HORACE S. UHLER. New York: D. Van Nostrand Co., 1928. Pp. 139. Figs. 31. $3.00.

This book consists of ten lectures on modern atomic physics given at the University of Vienna in 1926 for an audience comprising all faculties. The lectures begin with a treatment of the law of constant proportion and the law of multiple proportion, showing the existence of atoms as a necessary and inevitable consequence from a consideration of the experimental facts in chemistry. In the tenth lecture, the most recent growth of atomic physics, the wave-mechanics of Schroedinger, and the quantum-dynamics of Heisenberg are discussed. It is a long story of elaborate theories and painstaking experiments that bridges these two stages of the development. By touching upon the prominent features and high-lights, the intermediate lectures have succeeded in presenting a bird's-eye view of the whole progress.

The lectures are all non-mathematical and popular in nature as the circumstances required. In order to satisfy the intellectual curiosity of lay but highly educated and intelligent public, the basic experimental facts and the logical reasonings, demanding revolutionized theories, have been clearly demonstrated. Even readers without previous acquaintance with the subject will get the impression of being initiated into the secret of this most fascinating subject of atomic physics, of which the comparatively important advances have all been made in the present century.

Among the many interesting topics included in these lectures there may be mentioned the measurement of the smallest elementary charge of electricity; the determination of the velocity of light; the separation of elements; the isotopes, having exactly the same chemical property but different atomic weight; the picture of the infinitesimal atom as a planetary system, with all the intricacy of rotations and revolutions; and the transformations of the elements through the radioactive processes. The text is illustrated with pictures of the tracks of alpha particle, and photographs by Laue's method showing the regular arrangement of the atoms, etc.

Y. C. CHANG